



Remittances, Lorenz Dominance and Redistribution

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Abstract: We analyze how changes in the distribution of income, characterized by the comparison of Lorenz curves, affect public redistribution for an economy with international interfamily transfers (remittances). Our analysis suggests that a fall in the inequality of income might increase or reduce the government's ability to collect tax revenue and its electoral costs from inefficient taxation which in turn affect public redistribution. The main contribution of this paper is to characterize conditions in which a shift towards a dominant Lorenz curve can lead to an increase or fall in public redistribution. We also find that the composition of a change in the distribution of income, promoted by a change in the distribution of labor income or remittances, leads to different effects on the size of public redistribution.

JEL classification: F24, H23, D63, D72

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1 Introduction

In this paper we are interested in studying the effect of a change in the distribution of income on public redistribution for an economy in which there is a market based mechanism for income redistribution. In our economy, nationals living abroad send international private transfers (remittances) to a member of his (her) family who is living in the donor's home country. Remittances are motivated by altruism and represent a market based mechanism to redistribute income among members of a family. In addition, the government redistributes income throughout a linear tax-transfer program.

In this context, we are interested in studying how changes in the distribution of income modify public redistribution. Even though the topic of changes in the distribution of income and redistribution has been analyzed extensively in the literature, see Romer (1975), Roberts (1977), Meltzer and Richards (1981), Lambert (2001), Roemer (2001) among many others. Our analysis is different from the existing literature since we study the political calculus of public redistribution focusing on how changes in private transfers affect: i) The distribution of private income (defined as the sum of labor income plus private transfers) which in turn affects the electoral gains and costs from public redistribution, ii) The household's behavioral responses to the government's tax and spending policies since the government not only needs to consider the effect of taxes and spending on nationals working in the home country but also on international interfamily private transfers, and iii) The government's ability to collect tax revenue since private transfers affects the government's tax base.

Another relevant distinction of this paper with respect the literature is that, to the best of our knowledge, we are the first to analyze the effect of changes in the composition of the distribution of income on public redistribution.¹ In particular, we study whether a change of a full distribution of income towards a Lorenz dominant curve caused by changes in the distribution of labor income induces a different response from the government's redistributive program compared with a change in the distribution of income caused by remittances. Our analysis is developed for two cases: first, when the government uses a tax structure based on wage income and, second, when the tax structure collects public revenue from private income (the sum of labor income plus private transfers).

¹ There is a large literature of the possible crowding out effect of public transfers over interfamily transfers and charitable contributions (see Cox and Jimenez 1990 and Juárez 2009, among many others). However, to the best of our knowledge, there is little research on how international interfamily private transfers affect the incentives of the government to redistribute income (for early work on this issue see Kochi and Ponce 2010, 2011).

To study these issues, we develop a model of electoral competition in which parties select tax-transfer policies to maximize votes in the election. In our economy a change in the Lorenz curve affects the government's redistributive policy through two different channels: first, it affects the government's ability to collect tax revenue. Second, it modifies the distribution of the parties' electoral costs associated with the inefficiency costs of taxation. We also show that these two channels have differentiated effects on the government's political calculus of redistribution depending on whether the income tax is defined narrowly (over labor income) or more broadly (over labor income and private transfers). Hence our analysis sheds light on the role of exclusions (which define whether the tax base is narrow or broad) over public redistribution for an economy with private transfers. This is an issue that is relevant, from the perspective of policy design, for many countries that receive a significant amount of remittances such as China, India, México, etc.

The main findings of this paper are the following: first, for an economy with a tax structure characterized by a narrow definition of its tax base (when there is a tax on labor income and exclusions on private and public transfers), a fall in the inequality of income caused by a shift towards a dominant Lorenz curve in the distribution of labor income induces an increase (a fall) in the size of public redistribution if the covariance between labor income and the elasticity of the supply of labor and the income tax is negative (positive). This result could seem counterintuitive since we could expect that a fall in the inequality of labor income implies that the electoral gains for parties of implementing a linear tax-transfer system are lower (and therefore public redistribution should fall) since this kind of program redistributes income from higher than average labor income earners to lower than average labor income earners.

For this economy we also show that a change in the inequality of income explained by a change in the distribution of remittances does not modify the size of public redistribution since remittances do not affect the electoral costs associated with taxation and the electoral benefits from the government's public transfers. As a result, parties do not respond to a change in the distribution of income caused by remittances.

We extend our analysis to consider a proportional income tax applied to all sources of private income (labor income and private transfers) but exempts public transfers. In this case we find that the composition of changes in the distribution of income also matter since the response of the government's redistributive policy is different depending on whether income distribution changes because of a shock in the distribution of labor income or remittances. We identify further conditions in which a fall in the inequality of labor income and international private transfers induce an increase or a fall on public redistribution.

The rest of the paper is structured as follows: Section 2 contains the literature review. Section 3 incorporates the theoretical model and the electoral-economic equilibrium for a tax structure in which the income tax is applied only to labor income. Section 4 considers the case in which the

tax structure is constituted by a tax on private income (labor income and interfamily transfers). Section 5 concludes.

2 Literature Review

This paper is relevant to several brands of the economics literature: it is related to a large literature of the economic effects of remittances, to the political economy literature of public economics and to the analysis of altruism in the design of the government's tax and spending policies. According to the migration and development brief from the World Bank (2015), remittances play an important economic role in developing countries, such as India, China, México, etc. The literature shows that remittances have significant effects on private consumption, savings, the balance of payments, the distribution of income, among other economic outcomes (see also the report from OECD 2006).

Of particular interest for this paper is the effect of remittances on the distribution of income. On this issue, several papers suggest that remittances reduce the inequality in the distribution of income, however, there are papers that find the opposite. For instance, Acosta et al. (2007) find that remittances diminished the inequality in the distribution of income in Latin American and Caribbean countries. Adams and Page (2005) also find that remittances significantly reduced the level of poverty in developing countries. However, Stark et al. (1986, 1988) find that remittances might not only reduce inequality but also increase it if remittances are predominantly received by high income families.

The literature of the political economy of public finance basically rejects the idea that governments are controlled by benevolent social planners and that the government's tax and spending policy are not influenced by elections, electoral competition, and other political institutions. Within this context, the median voter model is the leading paradigm to study the role that political competition plays to shape economic policy decisions (see Downs 1957). Some of the most influential applications of the theory of elections to public finance include Romer (1975), Roberts (1977), Meltzer and Richards (1981), and Roemer (2001) who analyze an income tax-transfer program from the government. More recent analysis of electoral competition are the probabilistic voting models of elections (which are different from the median voter model since the latter models relax the assumption that parties have perfect information on the distribution of the voters' preferences) see Hettich and Winer (1999) and Coughlin (1992).

Finally, the literature on the role of altruism in the design of tax policy has focused, among other things, on the rationale of government intervention, see Hochman and Rodgers (1969), on how different types of altruistic behavior affect optimal Pigouvian taxes, see Johansson (1997), on the implications for the form of public transfers vis-a-vis in-kind transfers, see Coate (1995), and on the existence of majority rule equilibria that involves progressive taxation, see Kranich (2001).

In our economy, households receive international private transfers (or remittances) hence our paper is related to the economic effects of remittances in the economy. From this literature, our paper is more closely related to Kochi and Ponce (2010, 2011) who study the impact of remittances on the government's redistributive policy. Our analysis is different from Kochi and Ponce (2010, 2011) since they do not analyze how changes in the composition of the distribution of income could affect public redistribution. They also do not study the role of the tax structure, as we do in this paper, in determining the government's response to changes in the distribution of income. This is an issue that has not received adequate attention in the literature and that is relevant for policy design for many countries that receive a significant amount of remittances such as China, India, México, etc.

Finally, Kochi and Ponce (2010, 2011) do not compare two full distributions of income but they study the effect of changes in the distribution of income motivated by an exogenous change in income of donors of remittances. As we mentioned before, our paper studies the effect of changes in the distribution of income characterized by the comparison of Lorenz dominance on the size of public redistribution. Since many empirical papers use the Gini index to analyze the role of inequality on the government's policy, and the comparison of two distributions of income through dominant (dominated) Lorenz curves leads to unambiguous changes in the Gini index, then the analysis of this paper is more robust than previous analysis of this issue in the literature and provides tests that are more relevant for empirical analysis.

3 Political Equilibrium, Remittances and Redistribution with a Tax Structure Based on Labor Income

In this section we develop a voting model to analyze the interaction between international private transfers and the government's tax and spending policies. In particular, we are interested in analyzing how different tax structures imply different channels throughout remittances affect the government's redistributive policy. In our model we consider two types of households. National individuals living in the home country constitute the first type. Their utility function is $U(c, \ell)$ where preferences are strictly quasi-concave, c is consumption and $\ell \in [0, 1]$ is the supply of labor.

The households' labor earnings are heterogeneous and given by the distribution $h(w) > 0$: $\int_{w_0}^{w_{max}} h(w) = 1$ where w is the household's wage and $w \in [w_0, w_{max}]$.

The budget constraint is $c = w\ell(1 - \tau) + R + T$ where consumption depends on labor income $z = w\ell$, a tax τ on labor income, remittances R , and a public transfer T .

The second type of family is also a national household who works abroad and sends remittances to relatives in their home country. We call this type of family donors. We assume that donors of remittances are altruistic and care about the wellbeing of their relatives living in their home country.² Donors of interfamily private transfers have also strict quasi-concave preferences given by $U^a(c^a, \ell^a, v(\tau, T, w, R))$ where c^a is consumption of a family living abroad, $\ell^a \in [0,1]$ is the supply of labor with $\partial U^a / \partial \ell^a < 0$, and $v(\tau, T, w, R)$ is the indirect utility of those families who receive remittances. The assumption $\partial U^a / \partial v > 0$ means that there is interdependence of the utility functions of donors and households receiving remittances, and the wellbeing of a donor increases when the welfare of the family receiving remittances increases. The budget constraint for donors is given by $c^a = w^a \ell^a - R$ where w^a is the earning ability of households living abroad with $w^a \in [w_0^a, w_{max}^a]$. For simplicity we don't consider into our analysis the role of international taxes and transfers for nationals living abroad. We also assume that nationals living abroad do not pay taxes on income earned abroad.

Following Kochi and Ponce (2010, 2011), we assume households have no mobility. This assumption does not imply that the bi-causality role of migration and economic policy is not important but we assume this for mathematical simplicity. Hence, we take as given both the distribution of donors living abroad and the distribution of households receiving remittances in the home country.

In this economy there are two parties denoted by $i = 1,2$ that seek to win the election and form the government. Parties compete for votes by selecting a public transfer $T^i \forall i = 1,2$ financed through a tax on labor income $\tau^i \forall i$. The heterogeneity of labor income of households leads to conflicts among voters over the government's tax-transfer policy. There are voters whose ideal spending policy includes a high level of public redistribution while there are voters who would like the lowest level of public redistribution possible. These differences in the voters' demands over the government's public spending are solved throughout an election.

Parties run in the election with a platform over the size of public redistribution and voters vote to elect a party and to implement a policy platform. We follow the literature on linear income taxes, see Romer (1975), Roberts (1977) and Meltzer and Richards (1981) and consider a democracy with a single unit of government and a majoritarian electoral system in which the winner takes all. The party with a simple plurality of the votes is elected and implements its policy over taxes and redistribution.

² The literature considers mainly altruism, see Becker (1984), and economic exchange, see Bernheim et al. (1985), as rationales for interfamily transfers. We choose altruism as the rationale of interfamily transfers because there is significant theoretical and empirical attention in the literature to the phenomenon of altruism.

The structure of the political game that determines the formation of government is as follows: in the first stage of the game parties select tax and transfer policies to maximize the party's expected proportion of the vote in the election. In the second stage, voters observe the parties' policies and vote for the party that advances the tax-transfer policy that is closest to the voter's own views on the government's policy. All voters vote and voting is sincere. In the third stage of the game, votes are counted and the party with the largest number of votes forms the government and implements its policy platform.

In the second stage voters vote. The electoral calculus of a voter with a wage w is $\chi^i(w) = v(\tau^i, T^i, w, R) - v(\tau^{-i}, T^{-i}, w, R) \quad \forall w \in [w_0, w_{max}]$ where $\chi^i(w)$ reflects a rational choice of the vote and $v(\tau^i, T^i, w, R)$ is the welfare of voter with wage w if party i wins the election and implements policies τ^i, T^i while $v(\tau^{-i}, T^{-i}, w, R)$ is the voter's well being if the competing party $-i$ wins the election and implements policies τ^{-i}, T^{-i} .³

From the perspective of parties, voting is probabilistic.⁴ Hence the probability that a voter type w votes for party i , given tax and transfer policies τ^i, T^i and τ^{-i}, T^{-i} , is $F^i(\chi^i(w))$ where F^i is a continuous cumulative distribution function over $\chi^i(w)$. The aggregate expected proportion of votes for party i is given by $\phi^i = \int_{w_0}^y h(w) F^i(\chi^i(w)) dw$ $\forall y \in [w_0, w_{max}]$ and $\frac{\partial F^i(\chi^i(w))}{\partial \chi^i} = f^i(\chi^i(w))$ is the probability distribution function.

We follow the literature, see Coughlin (1999), Roemer (2001) and Mueller (2003), and assume ϕ^i is a strictly concave function of τ^i, T^i . The budget constraint of the government is $T^i = \tau^i \int_{w_0}^y h(w) w \ell(\tau^i, w) dw$ $\forall y \in [w_0, w_{max}]$ where $\tau^i \int_{w_0}^y h(w) w \ell(\tau^i, w) dw$ is the government's tax revenue.

Definition 1. *The electoral-economic equilibrium is constituted by weakly dominant policy strategies for parties $\tau^{*i}, T^{*i} \quad i = 1, 2$ and voting choices for households such that*

1.i) *In the first stage parties select:*

³ The notation $i, -i$ means that if $i = 1$ then party i is party 1 and party $-i$ is party two.

⁴ This assumption means that parties have imperfect information on the voters' preferences or parties do not know with certainty some parameters that determine the voters' choice of the vote, see Coughlin (1999) and Mueller (2003) for literature reviews on this issue.

$$\tau^{*i}, T^{*i} \in \operatorname{argmax} \phi^i = \int_{w_0}^y h(w) F^i(\chi^i(w)) dw \quad \forall i = 1, 2 \quad (1)$$

$$\text{s.t.} \quad T^i = \tau^i \int_{w_0}^y h(w) w \ell(\tau^i, w) dw \quad \forall y \in [w_0, w_{max}] \quad (2)$$

1.ii) In the second stage voters type $w \in [w_0, w_{max}]$ vote for party i if ⁵

$$\chi^i(w) = v(\tau^i, T^i, w, R) - v(\tau^{-i}, T^{-i}, w, R) > 0 \quad (3)$$

Otherwise, they vote for party $-i$

1.iii) In the third stage votes are counted. Consider Ω as a non-decreasing cumulative distribution of the sequence $\{\chi^i(w)\}_{\forall w}$. Therefore, if there exists a majority of voters $\forall w \in [w_0, w_{max}]: \chi^i(w) > 0$ then the following is satisfied

$$\Omega(\forall w \in [w_0, w_{max}]: \chi^i(w) > 0) > 1/2 \quad (4)$$

In this case party i wins the election and policies τ^{*i}, T^{*i} are implemented at the end of the third stage. Otherwise, party $-i$ wins the election and policies τ^{*-i}, T^{*-i} are implemented.

The electoral-economic equilibrium in definition 1 characterizes weakly dominant strategies for parties and households-voters. Parties select a tax and transfer policy taking into account the effect of their policies in the welfare of households and its impact in their choice of the vote. At the equilibrium, all households are maximizing their utility subject to their budget constraints and choosing their vote for the policy that is closer to their own views about the government's tax and spending policies.

3.2 The Government's Transfer Policy

In this section we characterize the parties' strategies for transfer policies.⁶ On what follows Lemma 1 shows the optimal platforms of parties on T^{*i} , proposition 1 provides a characterization that shows the links between the generalized Lorenz curve and T^{*i} , and proposition 2 shows a comparative static analysis of two Lorenz curves on the size of T^{*i} . For the analysis that follows it is convenient to define the elasticity of

⁵ If $\chi^i(w) = 0$ voters uses a fair coin to decide their vote.

⁶ We omit the characterization of τ^{*i} since the budget constraint of the government and T^{*i} are sufficient to fully specify the parties' policy platforms.

labor supply and the income tax for a voter type w as $\varepsilon_{\ell-\tau^i} = \frac{\partial \ell}{\partial \tau^i} \frac{1}{\ell}$.⁷ Moreover, we define $\sigma [f^i(\chi^i(w))\alpha, z(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and labor income $z(w) = w\ell$, where $f^i(\chi^i(w))$ is the marginal probability of the vote for party i and α is the voter's marginal utility of income, hence $f^i(\chi^i(w))\alpha$ approximates the electoral cost from voter type w if party i takes away \$1 through taxes. We also define $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ as the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$.

Households' average private income is $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ where $I(w) = w\ell + R$ is the sum of labor income $w\ell$ and private transfers R of voters with wage w , the term $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$ approximates the average gain in the parties' proportion of votes when the government transfers \$1 to voters throughout the redistributive program, and $\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw \forall y \in [w_0, w_{max}]$ is the economy's aggregate elasticity of the supply of labor-income tax.

To analyze the effect of changes in the distribution of income on T^{*i} we follow Lambert (2001) and define the generalized Lorenz curve as follows: $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in [0,1]$ which satisfies $p(y) = \int_{w_0}^y h(w)dw \forall y \in [w_0, w_{max}]$ such that the Lorenz curve of labor income-private income is $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_z dw \forall y \in [w_0, w_{max}]$ where $\Phi_{z-I} = \frac{z(w)}{\mu(I(w))}$ satisfying $L_{z-I}(p) \in [0,1]$ (see figure 1).^{8,9} Hence the generalized Lorenz curve of labor income-private income defined by $GL_{z-I}(p)$ is

$$\forall p \in [0,1]: p(y) = \int_{w_0}^y h(w)dw \Rightarrow GL_{z-I}(p) = \int_{w_0}^y h(w)z(w)dw = \mu(I(w))L_{z-I}(p) \tag{5}$$

⁷ Elasticity $\varepsilon_{\ell-\tau^i} = \frac{\partial \ell}{\partial \tau^i} \frac{(1-\tau^i)}{\ell}$ is evaluated at the point in which $\tau^i = 0$.

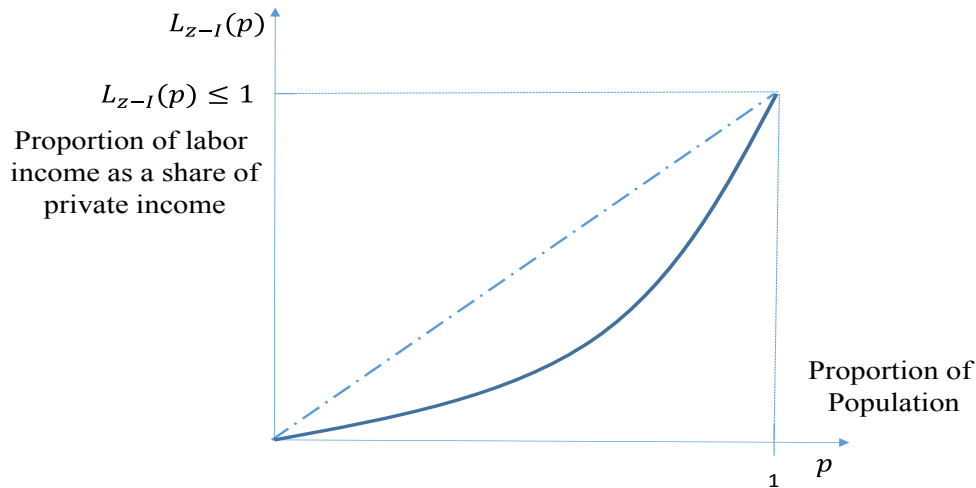
⁸ Since $\Phi_{z-I} = \frac{z(w)}{\mu(I(w))}$ and $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ where $I(w) = w\ell + R$ then $L_{z-I}(p) \leq 1$ with the limiting case of $L_{z-I}(p) = 1$ when $R = 0$ for all households.

⁹ For our analysis we use the generalized Lorenz curve (instead of the Lorenz curve) because it can allow comparisons of distributions of income for economies with different average incomes while meaningful comparisons of Lorenz curves usually requires to keep constant the economy's average income.

To analyze changes in the distribution of income induced by changes in the distribution of remittances define $\Phi_{R-I} = \frac{R}{\mu(I(w))}$. Hence, $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in [0,1]$ which satisfies $p(y) = \int_{w_0}^y h(w)dw$. The Lorenz curve of remittances-private income is $L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw \forall y \in [w_0, w_{max}]$. Hence, the generalized Lorenz curve of remittances-private income is $GL_{R-I}(p)$:

$$\begin{aligned} \forall p \in [0,1]: p(y) = \int_{w_0}^y h(w)dw &\Rightarrow GL_{R-I}(p) = \int_{w_0}^y h(w)R(w)dw \\ &= \mu(I(w))L_{R-I}(p) \end{aligned} \tag{6}$$

Figure 1. The Lorenz Curve for Labor Income $L_{z-I}(p)$



For the characterization of the equilibrium, we also define a generalized Lorenz curve for the distribution of votes, or electoral influence, as follow $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in [0,1]$ which satisfies $p(y) = \int_{w_0}^y h(w)dw$ such that the “Lorenz” curve of electoral influence is given by $L_{f^i}(p) = \int_{w_0}^y h(w)\Phi_{f^i}dw \forall y \in [w_0, w_{max}]$ where $\Phi_{f^i} = \frac{f^i(\chi^i(w))\alpha}{\mu(f^i(w))}$ and $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$. Therefore, the generalized “Lorenz” curve of electoral influence, $GL_{f^i}(p)$, is $\forall p \in [0,1]$:

$$\begin{aligned} p(y) = \int_{w_0}^y h(w)dw &\Rightarrow GL_{f^i}(p) = \int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw \\ &= \mu(f^i(w))L_{f^i}(p) \end{aligned} \tag{7}$$

Lemma 1. For an economy with a labor income tax system τ^{*i} and a universal per capita transfer T^{*i} , the politically optimal size of $T^{*i} \forall i = 1,2$ is given by:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w)z(w)dw - \frac{\int_{w_0}^y h(w)f^i(\chi^i(w))\alpha z(w)dw}{\int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw}}{-\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i} z(w)dw} \right\} \int_{w_0}^y h(w)z(w)dw \quad (8)$$

Proof. See the appendix.

Lemma 1 says that the politically optimal size of $T^{*i} \forall i = 1,2$ depends positively of the average labor income $\int_{w_0}^y h(w)z(w)dw$ (because an increase in the average labor income increases the government's marginal tax revenue for any given tax rate), negatively on a weighted average labor income $\frac{\int_{w_0}^y h(w)f^i(\chi^i(w))\alpha z(w)dw}{\int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw}$ (an increase on the weighted average income means that the electoral costs of taxation are higher and the government's fiscal incidence is lower which in turn leads to a lower equilibrium level of T^{*i}), and negatively on the inefficiency costs from income taxation given by a weighted average aggregate elasticity of the labor supply and the income tax $\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i} z(w)dw$ (higher inefficiency costs from taxation also increase the electoral costs associated with taxation and reduce the equilibrium level of the government's transfers).¹⁰

Since our interest is to analyze the effect of changes in the distribution of income in the per capita transfers from the government then proposition 1 express $T^{*i} \forall i = 1,2$ in terms of generalized Lorenz curves of labor income. Formally,

Proposition 1.

Define:

1.1) $\sigma [f^i(\chi^i(w))\alpha, z(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and $z(w)$;

¹⁰ Note that in the term $\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i} z(w)dw$ the labor supply-income tax elasticity of voter type w , $\varepsilon_{\ell-\tau^i}$, multiplies $z(w)$ and therefore $z(w)$ could be considered as the weight attached to $\varepsilon_{\ell-\tau^i}$ in the weighted aggregate elasticity of the labor supply and the income tax.

1.2) $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ as the households' average private income;

1.3) $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw$ as the Lorenz curve of labor income;

1.4) $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ as the covariance between the elasticity of the labor supply and income taxes of a household with wage w , $\varepsilon_{\ell-\tau^i}$, and $z(w)$ which is the household's labor income;

1.5) $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$ as the average marginal expected proportion of the votes that can be gained by providing a public transfer of \$1 to voters;

1.6) Define $\Phi_{f^i} = \frac{f^i(\chi^i(w))\alpha}{\int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw}$: $L_{f^i}(p) = \int_{w_0}^y h(w)\Phi_{f^i}dw$ with $L_{f^i}(p)$ as a Lorenz curve of electoral influence;

1.7) $\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw$ as the economy's average elasticity of the labor supply and income taxes.

Therefore, the universal per capita transfer $T^{*i} \forall i = 1,2$ can be expressed as follows:

$$T^{*i} = \frac{-\sigma[f^i(\chi^i(w))\alpha, z(w)]\mu(I(w))L_{z-I}(p)}{\{-\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w))L_{z-I}(p)\}\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw\}\{\mu(f^i(w))L_{f^i}(p)\}} \quad (9)$$

Proof. See the appendix.

Since $\mu(I(w))L_{z-I}(p) > 0$ and $\mu(f^i(w))L_{f^i}(p) > 0$, proposition 1 says that if: $-\sigma[f^i(\chi^i(w))\alpha, z(w)] > 0$ and $\{-\sigma[\varepsilon_{\ell-\tau^i}, \Phi_z] - \{\mu(I(w))L_{z-I}(p)\}\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw\} > 0$. Then $T^{*i} > 0$.

For this case, the size of public redistribution depends on the parties' electoral costs associated with the distribution of welfare costs from taxation, the distribution of political costs associated with the inefficiency costs of taxation, the distribution of electoral influence of the different coalitions of voters in the electorate (how many votes different groups of voters can deliver in the election), and the government's ability to collect tax revenue from the labor income tax. We now proceed to explain in more detail the political determinants of public redistribution.

In our model, the parties' electoral costs associated with the distribution of welfare costs of taxation are those determined by the reduction in the

indirect utility of voters as a result of a marginal increase in the income tax τ^{*i} .¹¹

These electoral costs are determined in condition (9) by $\sigma \left[f^i \left(\chi^i(w) \right) \alpha, z(w) \right]$ which is the covariance between $f^i \left(\chi^i(w) \right) \alpha$ which is the loss in the marginal probability of the vote from voter type w if party i takes away \$1 through the labor income tax, and the voter's labor income $z(w)$. If $-\sigma \left[f^i \left(\chi^i(w) \right) \alpha, z(w) \right] > 0$ then an increase of the covariance between $f^i \left(\chi^i(w) \right) \alpha$ and $z(w)$ implies higher welfare and electoral costs from the government's tax policy and as a result T^{*i} should be lower at the equilibrium.

Moreover, in (9) the expression $\mu(I(w))L_{z-I}(p)$ captures the effect of changes in the distribution of income on the government's ability to collect tax revenue. Condition (9) says that an increase in $L_{z-I}(p)$ leads to a higher collection of tax revenue for any given income tax rate (which implies a higher size of T^{*i} at the equilibrium) if the new distribution of income leads to a higher average of the household's private income $\mu(I(w))$.

The implementation of the labor income tax entails inefficiency costs from taxation which in turn lead to electoral costs that induce parties to limit the size of public redistribution.¹² The elasticity of the supply of labor and the income tax $\varepsilon_{\ell-\tau^i}$ measures how taxes distort the household's supply of labor. We follow the literature (see Blundell and MaCurdy 1998) in considering that a higher τ^i reduces the net return from labor services and promotes leisure which means that $\varepsilon_{\ell-\tau^i} < 0$.

In our model, the expressions that reflects the inefficiency costs from taxation are the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$, $\sigma \left[\varepsilon_{\ell-\tau^i}, z(w) \right]$, and the terms $\{ \mu(I(w))L_{z-I}(p) \} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw$ and $\mu(f^i(w))L_{f^i}(p)$. If $-\sigma \left[\varepsilon_{\ell-\tau^i}, z(w) \right] > 0$ then an increase in the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$ leads to higher inefficiency costs from taxation and higher political costs that imply a lower level of T^{*i} .

Proposition 1 also says that a change in the distribution of income modifies the distribution of electoral costs from inefficient taxation. This effect is captured in our model by the product between the generalized Lorenz curve of labor income-private income and the economy's aggregate elasticity of the supply of labor and the income tax

¹¹ An increase in τ^{*i} , ceteris paribus, reduces the household's ability to consume private goods and leisure which in turn leads to a fall in the household's indirect utility.

¹² The electoral costs from inefficient taxation are the votes lost from the inefficiencies created by the labor income tax that distorts the household's relative prices between leisure and consumption of the private good.

$\{\mu(I(w))L_{z-l}(p)\} \int_{w_0}^y h(w) \varepsilon_{\rho-\tau} dw$. Following our assumption that $\varepsilon_{\rho-\tau} < 0$ then $-\{\mu(z(w))L_{z-l}(p)\} \int_{w_0}^y h(w) \varepsilon_{\rho-\tau} dw > 0$. In this case, the higher this term the higher the political costs from economic distortions caused by taxation and the lower is T^{*i} at the political equilibrium. Finally, the electoral costs for inefficient taxation are also positively related to the distribution of electoral influence of different coalitions of voters in the electorate. This effect is captured by the generalized Lorenz curve of electoral influence $\mu(f^i(w))L_{fi}(p)$.

Of particular interest for this paper is the effect of changes in the distribution of income on the size of T^{*i} . We argue that a reduction in the inequality in the distribution of labor income modifies the parties' platforms on income redistribution in the following ways: first, it changes the government's ability to collect tax revenue. Given a tax system, a fall in the inequality of income might be accompanied by an increase, a fall or a constant tax revenue for the government if the new distribution of income implies a higher, lower, or equal average in the household's labor income. If the government's tax revenue increases (declines) as a result of a fall in the inequality in the distribution of income then this effect tends to increase (reduce) T^{*i} . Second, a fall in the inequality of income might lead to a higher, lower, or an equal distribution of the inefficiency costs from taxation. However, if the electoral costs from inefficient taxation increase (decrease) as a result of a fall in the inequality of labor income then T^{*i} should be lower (higher) at the political equilibrium.¹³

From our previous discussion, it is obvious that a reduction in the inequality of the distribution of income might lead to many possible results on T^{*i} since the different effects mentioned above might affect T^{*i} in ways that may oppose each other. On what follows proposition 2 makes a precise statement about the impact of a reduction in the inequality of labor income on the size of T^{*i} .

¹³ A third effect, is that a change in the distribution of income affects the households' opportunity costs of taking away \$1 through the tax system and hence it modifies the distribution of welfare costs for voters and the parties' overall electoral costs from taxation. If a fall in the inequality of distribution of income reduces (increases) the opportunity costs of households of taking away \$1 through the tax system then T^{*i} will be higher (lower). However, in the following propositions we are interested in studying how a change in the dominance of the Lorenz curve affects T^{*i} . Hence, the calculation of the derivative of T^{*i} with respect $L_{z-l}(p)$ means that we have to impose a *ceteris paribus* condition that implies that a change in the Lorenz curve do not affect the covariance between $f^i(\chi^i(w))\alpha$ and $z(w)$, that is $\sigma[f^i(\chi^i(w))\alpha, z(w)]$ remains unchanged, and as a result the distribution of the voters' welfare costs of taxation do not change.

For the comparative analysis between two distributions of income, we follow Lambert (2001) and assume two distributions $z(w)$ and $\hat{z}(w)$ leading to two different generalized Lorenz curves $GL_{z-1}(p)$ and $\widehat{GL}_{z-1}(p)$ such that the distribution $\hat{z}(w)$ Lorenz dominates $z(w)$ implying $\widehat{GL}_{z-1}(p) \geq GL_{z-1}(p) \quad \forall p \in [0,1]$ (see figure 2 in the appendix for a representation of how the distribution of income $\hat{z}(w)$ Lorenz-dominates the distribution $z(w)$).¹⁴ We also denote $T^{*i} \Big|_{\widehat{GL}_{z-1}(p)}$ as the size of the government's per capita transfer at the political equilibrium under distribution of income $\widehat{GL}_{z-1}(p)$ and $T^{*i} \Big|_{GL_{z-1}(p)}$ under $GL_{z-1}(p)$.

Proposition 2.

Assume two distributions $z(w)$ and $\hat{z}(w)$ leading to generalized Lorenz curves $GL_{z-1}(p) = \mu(I(w))L_{z-1}(p)$ with $L_{z-1}(p) = \int_{w_0}^y h(w)\Phi_{z-1}dw$ and $\widehat{GL}_{z-1}(p) = \mu(I(w))\widehat{L}_{z-1}(p)$ with $\widehat{L}_{z-1}(p) = \int_{w_0}^y h(w)\widehat{\Phi}_{z-1}dw \quad \forall y \in [w_0, w_{max}]$ such that the distribution $\hat{z}(w)$ Lorenz-dominates $z(w)$ implying $\widehat{GL}_{z-1}(p) \geq GL_{z-1}(p) \quad \forall p \in [0,1]$.

If $\sigma[\varepsilon_{\ell-t^i}, z(w)] \stackrel{\leq}{>} 0$ then

$$\widehat{GL}_{z-1}(p) \geq GL_{z-1}(p) \implies T^{*i} \Big|_{\widehat{GL}_{z-1}(p)} \stackrel{>}{<} T^{*i} \Big|_{GL_{z-1}(p)} \tag{10}$$

Proof

$$\text{If } \frac{\partial T^{*i}}{\partial L_{z-1}(p)} \stackrel{\geq}{<} 0 \iff \widehat{GL}_{z-1}(p) \geq GL_{z-1}(p) \quad \forall p \in [0,1]$$

$$\implies T^{*i} \Big|_{\widehat{L}_{z-1}(p)} \stackrel{\geq}{<} T^{*i} \Big|_{L_{z-1}(p)}. \tag{15}$$

From (8) it follows that

¹⁴ In this paper we will use the expression “a change towards a dominant Lorenz curve” to describe a shift from a Lorenz curve $GL_{z-1}(p)$ to the new Lorenz curve $\widehat{GL}_{z-1}(p)$ such that $\widehat{GL}_{z-1}(p) \geq GL_{z-1}(p) \quad \forall p \in [0,1]$.

¹⁵ Note that $\frac{\partial T^{*i}}{\partial GL_{z-1}(p)} = \mu(I(w)) \frac{\partial T^{*i}}{\partial L_{z-1}(p)}$ since $\mu(I(w)) > 0$, $sign \left(\frac{\partial T^{*i}}{\partial L_{z-1}(p)} \right)$

$$\implies sign \left(\frac{\partial T^{*i}}{\partial GL_{z-1}(p)} \right).$$

$$\begin{aligned} \frac{\partial T^{*i}}{\partial L_{z-i}(p)} &= \\ &= \frac{T^{*i}}{L_{z-i}(p)} + \frac{T^{*i} \mu(I(w)) \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw}{\left\{ -\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w))L_{z-i}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw \right\}} \end{aligned} \quad (11)$$

$$\begin{aligned} \Rightarrow \frac{\partial T^{*i}}{\partial L_{z-i}(p)} &= \\ &= \frac{-T^{*i} \sigma[\varepsilon_{\ell-\tau^i}, z(w)]}{L_{z-i}(p) \left\{ -\sigma[\varepsilon_{\ell-\tau^i}, \Phi_z] - \{\mu(I(w))L_{z-i}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw \right\}} \geq 0 \end{aligned} \quad (12)$$

Recall $T^{*i} > 0$, $\left\{ -\sigma[\varepsilon_{\ell-\tau^i}, \Phi_z] - \{\mu(I(w))L_{z-i}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw \right\} > 0$,

and $L_{z-i}(p) > 0$, hence $\sigma[\varepsilon_{\ell-\tau^i}, z(w)] \leq 0 \Rightarrow \frac{\partial T^{*i}}{\partial L_{z-i}(p)} \geq 0$.

In proposition 2 we identify a condition that allows us to state unambiguously the impact of a change towards a *dominant* generalized labor-private income *Lorenz curve*, on the size of the per capita public transfer T^{*i} . In particular if $\sigma[\varepsilon_{\ell-\tau^i}, z(w)] \leq 0$ then:

$$\widehat{GL}_{z-i}(p) \geq GL_{z-i}(p) \Rightarrow T^{*i} \Big|_{\widehat{GL}_{z-i}(p)} > T^{*i} \Big|_{GL_{z-i}(p)}$$

As we mentioned before, a fall in the inequality in the distribution of labor income has an effect on T^{*i} through two different channels: the first effect modifies the government's ability to collect tax revenue and the second effect the distribution of electoral costs associated with the inefficiency costs of taxation. To explain further our results, consider the case in which $\sigma[\varepsilon_{\ell-\tau^i}, z(w)] < 0$ then:

$$\widehat{GL}_{z-i}(p) \geq GL_{z-i}(p) \Rightarrow T^{*i} \Big|_{\widehat{L}_{z-i}(p)} \geq T^{*i} \Big|_{L_{z-i}(p)} > 0$$

that is, a fall in the inequality in the distribution of labor income leads to a higher size of the government's per capita transfer T^{*i} . This result seems counterintuitive since we could expect that a fall in the inequality of labor income implies that the electoral gains of implementing a linear tax-transfer system for parties are lower (and therefore T^{*i} should be lower) since this kind of program redistributes income from higher than average labor income earners to lower than average labor income earners.

This result is explained as follows: according to condition (11), a change towards a dominant generalized labor-private income Lorenz curve affects

positively the ability of the government to collect tax revenue which induces an increase in T^{*i} (this marginal effect is identified in condition 11 by the term $\frac{T^{*i}}{L_{Z-I}(p)}$). Condition (11) also says that an increase in $GL_{Z-I}(p)$ increases the parties' electoral costs which tend to reduce T^{*i} . To see this, note that condition (11) shows that the marginal change in the party's electoral costs associated with inefficient taxation is given by the

term $\frac{T^{*i}\mu(I(w))\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw}{\{-\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w))L_{Z-I}(p)\}\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw\}} < 0$ since:

$$T^{*i}\mu(I(w)) > 0, \int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw < 0$$

$$\text{and } \{-\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w))L_{Z-I}(p)\}\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw\} > 0$$

Proposition 2 says (see condition 12) that if $\sigma[\varepsilon_{\ell-\tau^i}, z(w)] < 0$, that is if individuals with higher than average labor income also display lower than average elasticities of the supply of labor and the income tax, then the tax revenue effect dominates the rise in the electoral costs from inefficient taxation and as a result T^{*i} is higher. If $\sigma[\varepsilon_{\ell-\tau^i}, z(w)] > 0$ the opposite occurs and a fall in the inequality of income increases in a sufficient way the electoral costs from inefficient taxation to cause a fall in T^{*i} .

Another outcome from proposition 2 is that a change in the distribution of income explained by a change in the distribution of remittances does not modify the size of public redistribution. This is explained by the fact that for an economy with a tax structure characterized by a tax on labor income and exclusions on private and public transfers, a fall in the inequality of income caused by changes in remittances does not affect the electoral costs associated with taxation and the electoral benefits from public redistribution. As a result, parties do not respond to a change in the distribution of income caused by remittances.

4 A Tax Structure Based on All Sources of Private Income and Redistribution

An alternative tax system of interest for governments is to consider a broader definition of the base of the income tax. In a tax system on private income a proportional tax τ^i is applied to all sources of private income, that is, τ^i collects public revenue from the sum of the households' labor income plus remittances but exempts public transfers. This type of tax structure is attractive to governments because an exemption of public transfers from income taxation improves the government's capacity to redistribute income among households-voters. In this section we analyze this case. For this economy, the budget constraint of a national household living in the home country is $c = (w\ell + R)(1 - \tau^i) + T^i$ where consumption depends on labor income $z = w\ell$, remittances R , and a

public transfer T^i . The government's budget constraint is $T^i = \tau^i \int_{w_0}^y h(w) \{ w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w) \} dw$.

As before, the electoral equilibrium characterizes weakly dominant strategies for parties and households for this economy. Parties select a tax-transfer policy taking into account the effect of their policies on the welfare and voting behavior of households in the home country and the impact of the government's tax-transfer policy on international private transfers.

Before we characterize the parties' strategies on policy design we analyze the marginal effect of an increase in τ^i in the size of remittances. This is relevant for tax-transfer policy design because the inefficiency costs of income taxation might not only be reflected as a fall in the overall size of remittances but as an increase in international private transfers. The sign of a change of remittances due to a marginal increase in τ^i is also relevant for the government's ability to collect tax revenue and the size of transfers.

In proposition 3 we show that an increase in τ^i has an ambiguous effect on remittances since a higher τ^i reduces the marginal benefit and the marginal cost of donors of sending remittances. To see this, note that donors of remittances decide the size of their private transfers by recognizing: first, an increase in the size of remittances increases directly the consumption and well-being of the family receiving the interfamily transfers. An increase in τ^i reduces the positive impact of remittances on consumption of the family receiving remittances (which, from the perspective of donors, is the marginal benefit of sending remittances) and therefore an increase in τ^i tends to reduce the size of remittances.

Second, remittances also affect the household's demand of leisure by increasing it or reducing it depending on whether leisure is a normal or an inferior good. If leisure is a normal good, an increase in remittances reduces the household's supply of labor, the household's net labor income and consumption which, in turn, reduces the welfare of the household receiving remittances. This effect is considered, from the point of view of donors, as the marginal cost of sending remittances. An increase in τ^i reduces the marginal costs of sending remittances and this effect tends to increase the size of remittances.

Thus, the net effect of an increase in τ^i on the size of remittances depends on whether the change in the marginal benefit outweighs the corresponding change in the marginal costs of sending remittances. On what follows proposition 3 finds sufficient conditions in which remittances increase or fall as a result of an increase in the private income tax τ^i .

Proposition 3.

The size of remittances sent by households living abroad is given by $R(\tau^i, T^i, w^a, w) \in \operatorname{argmax} \mu^a(c^a, \ell^a, v(\tau, T, w, R))$ s.t: $c^a = w^a \ell^a - R$.

Define $\varepsilon_{\ell-R} = \frac{\partial \ell}{\partial R} \frac{1}{\ell}$. Moreover,

$$\text{If } (\varepsilon_{\ell-R} w\ell + 1) \begin{matrix} < \\ > \end{matrix} 0 \text{ then } \frac{dR}{d\tau^i} \begin{matrix} > \\ < \end{matrix} 0 \quad (13)$$

Proof. See the appendix.

Proposition (3) quantifies the impact of an increase of τ^i on the size of remittances. As we mentioned before, an increase in τ^i has an ambiguous effect on remittances since it reduces the marginal benefit and cost of remittances therefore an increase in τ^i leads to an ambiguous effect on remittances. Proposition 3 says that the sign of $dR/d\tau^i$ depends on the value of the elasticity of the supply of labor and remittances. In particular, for $\varepsilon_{\ell-R} < 0$: $\varepsilon_{\ell-R} > \frac{-1}{w\ell}$ then $\frac{dR}{d\tau} < 0$ (see proposition 3). A marginal increase in τ^i leads to a lower size of remittances because the fall in the marginal benefit of remittances, as a result of a higher τ^i , outweighs the fall of its marginal cost and therefore the size of remittances falls. If $\varepsilon_{\ell-R} < 0$: $\varepsilon_{\ell-R} < \frac{-1}{w\ell}$, the opposite occurs and remittances increase with a marginal increase of τ^i , that is $\frac{dR}{d\tau^i} > 0$.

4.2 Public Transfers under a Tax Structure on All Sources of Private Income

In this section we characterize weakly dominant strategies of parties for tax and transfer policies. On what follows, Lemma 2 characterizes optimal policy platforms of parties when the tax structure collects tax revenue from private income (labor income and remittances) but exempts public transfers, and proposition 4 express $T^{*i} \forall i = 1,2$ in terms of generalized Lorenz curves of labor income and remittances. The analysis of this section allows us to say more about the role of the composition of changes in the distribution of income (that might be caused by shocks in the distribution of labor income vis-à-vis shocks in the distribution of remittances) on the size of public redistribution.

For the analysis that follows we define the remittances-income tax elasticity of a voter type w as $\varepsilon_{R-\tau^i} = \frac{\partial R}{\partial \tau^i} \frac{1}{R}$, the covariance between $f^i(\chi^i(w))\alpha$ and $I(w)$ is equal to $\sigma[f^i(\chi^i(w))\alpha, I(w)]$ where $I(w) = w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)$ is equal to the household's private income, $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ is the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$, and $\sigma[\varepsilon_{R-\tau^i}, R]$

is the covariance between $\varepsilon_{R-\tau^i}$ and R where R is the size of remittances of the household with private income $I(w)$.¹⁶

For the analysis of the distribution of income let define, Φ_{z-I} and Φ_{R-I} as follows $\Phi_{z-I} = \frac{z(w)}{\int_{w_0}^{w_{max}} h(w)I(w)dw}$ and $\Phi_{R-I} = \frac{R}{\int_{w_0}^{w_{max}} h(w)I(w)dw}$.¹⁷ Moreover consider that $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in [0,1]$ which satisfies $p(y) = \int_{w_0}^y h(w)dw \forall y \in [w_0, w_{max}]$. Hence, the Lorenz curve of labor income over private income is $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw \forall y \in [w_0, w_{max}]$ and the Lorenz curve for remittances over private income is $L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw \forall y \in [w_0, w_{max}]$.

Let $\mu(z(w)) = \int_{w_0}^{w_{max}} h(w)z(w)dw$ be the average labor income, $\mu(R(w)) = \int_{w_0}^{w_{max}} h(w)R(w)dw$ the average size of remittances and $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ the average private income. We also define $s_{z-I} = \mu(z(w))/\mu(I(w))$ and $s_{R-I} = \mu(R(w))/\mu(I(w))$ as the shares of labor income over private income and remittances over private income. To analyze the impact of the composition of changes in the distribution of income on T^{*i} , we define generalized Lorenz curves of labor income over private income and remittances over private income, and the Lorenz curve of electoral influence respectively by $GL_{z-I}(p), GL_{R-I}(p)$ and $GL_{fi}(p)$. Hence, $\forall p \in [0,1]$:

$$p(y) = \int_{w_0}^y h(w)dw \Rightarrow GL_{z-I}(p) = \int_{w_0}^y h(w)z(w)dw = \mu(I(w))L_{z-I}(p) \quad (14)$$

and

$$p(y) = \int_{w_0}^y h(w)dw \Rightarrow GL_{R-I}(p) = \int_{w_0}^y h(w)R(w)dw = \mu(I(w))L_{R-I}(p) \quad (15)$$

and

$$\begin{aligned} p(y) = \int_{w_0}^y h(w)dw \Rightarrow GL_{fi}(p) &= \int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw = \\ &= \mu(f^i(w))L_{fi}(p) \end{aligned} \quad (16)$$

Next, Lemma 2 characterizes politically optimal transfers $T^{*i} \forall i$ for an economy with a tax on private income and proposition 4 express

¹⁶ As before, the elasticities are evaluated at the point in which $\tau^i = 0$.

¹⁷ In the previous section we did not define Φ_{R-I} because changes in the distribution of remittances did not affect the parties' policy platforms.

$T^{*i} \forall i = 1,2$ in terms of generalized *Lorenz curves* of labor income and remittances.

Lemma 2. Assume a tax system with a personal income tax τ^{*i} on labor income and interfamily private transfers. For this economy $T^{*i} \forall i = 1,2$ is given by:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w)I(w)dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w)dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}}{-\left\{ \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w)dw + \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} Rdw \right\}} \right\} \int_{w_0}^y h(w)I(w)dw \quad (17)$$

Proof. See the appendix

Lemma 2 says that if the income tax is applied to labor income and interfamily private transfers then the politically optimal size of $T^{*i} \forall i = 1,2$ depends positively on the households' average private income $\int_{w_0}^y h(w)I(w)dw$ (because an increase in the average private income increases the government's marginal tax revenue for any given tax rate), negatively on a weighted average private income $\frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w)dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}$ (an increase on the weighted average private income means that the welfare and electoral costs of taxation are higher and the government's fiscal incidence is lower which in turn leads to a lower equilibrium level of T^{*i}), and negatively on the inefficiency costs from income taxation which are given by a weighted aggregate elasticity of the labor supply and income taxes $\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w)dw$ and a weighted aggregate elasticity of remittances and income taxes $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} Rdw$ (recall that the inefficiency costs of taxation lead to electoral costs which in turn reduce the equilibrium level of T^{*i}).¹⁸

¹⁸ As we have defined before, in the expression $\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w)dw$, the labor supply-income tax elasticity of voter type w , $\varepsilon_{\ell-\tau^i}$, multiplies $z(w)$ and therefore $z(w)$ could be considered as the weight attached to $\varepsilon_{\ell-\tau^i}$ in the weighted aggregate elasticity of the labor supply and income taxes. Similarly, in the expression $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} Rdw$, the remittance-income tax elasticity of voter type w , $\varepsilon_{R-\tau^i}$,

Proposition 4.

Define:

4.1) $\sigma [f^i(\chi^i(w))\alpha, I(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and the household's private income $I(w) = w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)$;

4.2) $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ as the average private income;

4.3) $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw$ as the Lorenz curve of labor income;

4.4) $L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw$ as the Lorenz curve of remittances;

4.5) $\Psi_{w\ell-\tau^i} = -\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - s_{z-I}\{\mu(z(w))L_{z-I}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw$, where $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ is the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$, $s_{z-I} = \frac{\mu(z(w))}{\mu(I(w))}$ is the share of average labor income and average private income, $\mu(z(w)) = \int_{w_0}^{w_{max}} h(w)z(w)dw$ is the average labor income, and $\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw$ is the economy's aggregate elasticity of labor supply and income taxes;

4.6) $\Psi_{R-\tau^i} = -\sigma[\varepsilon_{R-\tau^i}, R(w)] - s_{R-I}\{\mu(R(w))L_{R-I}(p)\} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw$, where $\sigma[\varepsilon_{R-\tau^i}, R(w)]$ is the covariance between $\varepsilon_{R-\tau^i}$ and R , $s_{R-I} = \frac{\mu(R(w))}{\mu(I(w))}$ is the share of average interfamily private transfers and average private income, $\mu(R(w)) = \int_{w_0}^{w_{max}} h(w)Rdw$ is the average size of remittances, and $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw$ is the economy's aggregate elasticity of interfamily private transfers and income taxes;

4.7) $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$ as the average marginal expected proportion of the votes that can be gained by providing a public transfer of \$1 to voters.

4.8) Define $\Phi_{f^i} = \frac{f^i(\chi^i(w))\alpha}{\int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw}$: $L_{f^i}(p) = \int_{w_0}^y h(w)\Phi_{f^i}dw$ with $L_{f^i}(p)$ as a Lorenz curve of electoral influence.

Therefore, the universal per capita transfer $T^{*i} \forall i = 1,2$ can be expressed as follows:

$$T^{*i} = \left\{ \frac{-\sigma [f^i(\chi^i(w))\alpha, I(w)]\mu(I(w))\{L_{z-I}(p) + L_{R-I}(p)\}}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}\{\mu(f^i(w))L_{f^i}(p)\}} \right\} \quad (18)$$

Proof. See the appendix.

multiplies R and therefore R could be considered as the weight attached to $\varepsilon_{R-\tau^i}$ in the weighted aggregate elasticity of interfamily private transfers and income taxes.

Proposition 4 says that since $\mu(I(w))\{L_{z-I}(p) + L_{R-I}(p)\} > 0$ and $\mu(f^i(w))L_{f^i}(p) > 0$, if $-\sigma [f^i(\chi^i(w))\alpha, I(w)] > 0$ and $\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} > 0$ then $T^{*i} > 0$. For economies with a tax structure on private income, changes in the distribution of labor income and remittances affect both: the government's ability to collect tax revenue and the distribution of the political costs associated with inefficient taxation.

To analyze further the impact of changes in the different components of income on T^{*i} , proposition 5 characterizes the effect of a fall in the inequality in the distribution of labor income on T^{*i} and proposition 6 of a fall in the inequality in the distribution of remittances on T^{*i} .

Proposition 5.

Assume two distributions $z(w)$ and $\hat{z}(w)$ leading to generalized Lorenz curves:

$$GL_{z-I}(p) = \mu(I(w))L_{z-I}(p) \text{ with } L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw$$

and

$$\widehat{GL}_{z-I}(p) = \mu(I(w))\widehat{L}_{z-I}(p) \text{ with } \widehat{L}_{z-I}(p) = \int_{w_0}^y h(w)\widehat{\Phi}_{z-I}dw \quad \forall y \in [w_0, w_{max}]$$

such that the distribution $\hat{z}(w)$ Lorenz-dominates $z(w)$ implying $\widehat{GL}_{z-I}(p) \geq GL_{z-I}(p) \quad \forall p \in [0,1]$. If:

$$\widehat{GL}_{z-I}(p) \geq GL_{z-I}(p) \implies T^{*i}|_{\widehat{GL}_{z-I}(p)} > T^{*i}|_{GL_{z-I}(p)} \tag{19}$$

Proof.

Consider condition (18). It follows that

$$\frac{\partial T^{*i}}{\partial L_{z-I}(p)} = \frac{T^{*i}}{\{L_{z-I}(p) + L_{R-I}(p)\}} + \frac{T^{*i}s_{z-I} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} > 0 \tag{20}$$

$$\text{Recall } T^{*i} > 0, \{L_{z-I}(p) + L_{R-I}(p)\} > 0 \implies \frac{T^{*i}}{\{L_{z-I}(p) + L_{R-I}(p)\}} > 0.$$

$$\text{Moreover, } \{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} > 0, s_{z-I} > 0 \text{ and } \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw < 0 \implies \frac{T^{*i}s_{z-I} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} < 0. \text{ As a result } \frac{\partial T^{*i}}{\partial L_{z-I}(p)} > 0.$$

Proposition 6.

Assume two distributions of remittances $R(w)$ and $\hat{R}(w)$ leading to generalized Lorenz curves:

$$GL_{R-I}(p) = \mu(I(w))L_{R-I}(p) \text{ with } L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw$$

and

$$\widehat{GL}_{R-I}(p) = \mu(I(w))\widehat{L}_{R-I}(p) \text{ with } \widehat{L}_{R-I}(p) = \int_{w_0}^y h(w)\widehat{\Phi}_{R-I}dw \forall y \in [w_0, w_{max}]$$

such that the distribution $\widehat{R}(w)$ Lorenz-dominates $R(w)$ implying $\widehat{GL}_{R-I}(p) \geq GL_{R-I}(p) \forall p \in [0,1]$. Moreover,

$$\begin{aligned} &\text{if } \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw \geq 0 \\ &\text{then } \widehat{GL}_{R-I}(p) \geq GL_{R-I}(p) \Rightarrow T^{*i}|_{\widehat{L}_{R-I}(p)} > T^{*i}|_{L_{R-I}(p)} \end{aligned} \tag{6.1}$$

$$\begin{aligned} &\text{if } \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw < 0 \\ &\text{then } \widehat{GL}_{R-I}(p) \geq GL_{R-I}(p) \Rightarrow T^{*i}|_{\widehat{L}_{R-I}(p)} \begin{matrix} \geq \\ < \end{matrix} T^{*i}|_{L_{R-I}(p)} \end{aligned} \tag{6.2}$$

Proof.

From (18), we can show that

$$\frac{\partial T^{*i}}{\partial L_{R-I}(p)} = \frac{T^{*i}}{\{L_{Z-I}(p) + L_{R-I}(p)\}} + \frac{T^{*i} s_{R-I} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} \tag{21}$$

Consider condition (21), if $\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} > 0, T^{*i} s_{R-I} > 0$ then $\text{sign}(\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw) \Rightarrow \text{sign}\left(\frac{T^{*i} s_{R-I} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}}\right)$.

Hence, $\frac{T^{*i}}{\{L_{Z-I}(p) + L_{R-I}(p)\}} > 0$ and $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw \geq 0 \Rightarrow \frac{\partial T^{*i}}{\partial L_{R-I}(p)} > 0$.

However, if $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw < 0$ then $\frac{\partial T^{*i}}{\partial L_{R-I}(p)} < 0$.

Propositions 5 and 6 show that a fall in the distribution of labor income and remittances have, in general, an ambiguous effect on T^{*i} . To be more specific, a change towards a dominant generalized Lorenz curve of labor-private income induces party i to increase T^{*i} due to a higher tax revenue collection and, simultaneously, a fall in T^{*i} due to an increase in the electoral costs from the inefficiencies on the households' supply of labor explained by the income tax. From the comparative analysis in condition (20), the full effect on T^{*i} of a change towards a dominant

Lorenz curve is given by $\frac{\partial T^{*i}}{\partial L_{Z-I}(p)} = \frac{T^{*i}}{\{L_{Z-I}(p) + L_{R-I}(p)\}} + \frac{T^{*i} s_{Z-I} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} \begin{matrix} \geq \\ < \end{matrix} 0$

where $\frac{T^{*i}}{\{L_{Z-I}(p) + L_{R-I}(p)\}} > 0$ represents an increase in the government's tax revenue as a result of a change towards a Lorenz dominant distribution of labor income while the term $\frac{T^{*i} s_{Z-I} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} < 0$, where $T^{*i} s_{Z-I} > 0$ $\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} > 0$ and $\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw < 0$, represents an increase in

the parties' electoral costs from inefficient labor income taxation.¹⁹ If the tax revenue effect dominates (is dominated by) the rise in the electoral costs from inefficient taxation then T^{*i} increases (falls).

For the case of proposition 6, the comparative analysis shows that the effect in T^{*i} due to a shift towards a dominant generalized Lorenz curve of remittances-private income is given by $\frac{\partial T^{*i}}{\partial L_{R-I}(p)} = \frac{T^{*i}}{\{L_{Z-I}(p)+L_{R-I}(p)\}} + \frac{T^{*i} S_{R-I} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} \geq 0$ where $\frac{T^{*i}}{\{L_{Z-I}(p)+L_{R-I}(p)\}} > 0$ represents an increase in the government's tax revenue as a result of a change towards a dominant Lorenz curve of remittances while the change in the parties' electoral costs from inefficient income taxation is given by the term $\frac{T^{*i} S_{Z-I} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}} \geq 0$ where $T^{*i} S_{Z-I} > 0$ $\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} > 0$ and $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw \geq 0$.

Hence, a fall in the distribution of income caused by a reduction in the inequality of remittances induces party i to increase T^{*i} due to a higher tax revenue collection but it also produces an ambiguous effect on T^{*i} due to the electoral costs from the inefficiencies on the remittances-income tax elasticity (characterized by the expression $\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw$) might be higher if this elasticity is negative, or lower if the elasticity is positive. As a result, proposition (6.1) says that if the remittances-income tax elasticity is positive then, unambiguously, a change towards a dominant Lorenz curve of remittances induces the government to increase T^{*i} . This is the case because a fall in the inequality of the distribution of remittances increases the government's tax revenue (which tends to increase T^{*i}) and it reduces the electoral costs from inefficient taxation that also tends to increase T^{*i} .

However, if the remittances-income tax elasticity is negative the electoral costs from inefficient taxation increase and induce parties to reduce T^{*i} . In this case, the change in T^{*i} depends on whether the tax revenue effect, which tends to increase T^{*i} , dominates (is dominated by) the rise in the electoral costs from inefficient taxation which effect tends to reduce T^{*i} .

5 Conclusions

In our economy, nationals living abroad send international private transfers (remittances) which represent a market based mechanism to redistribute income. In our analysis we study the impact of changes in the

¹⁹ The negative sign of the latter term means that a change towards a *dominant* generalized Lorenz curve $\widehat{GL}_{Z-I}(p)$ increases the parties' electoral costs from inefficient taxation and this effect tends to reduce T^{*i} .

composition in the distribution of income caused by changes in labor income and remittances on public redistribution. Our analysis is developed for two cases of interest: first, when the government uses a tax structure based on wage income and, second, when the tax structure collects public revenue from private income (the sum of labor income and interfamily private transfers). This is an issue that is relevant, from the perspective of policy design, for many developing countries that receive a significant amount of remittances.

For a tax structure characterized by a tax on labor income and exclusions on private and public transfers, a change in the distribution of labor income that leads to a dominant Lorenz curve, modifies the government's ability to collect tax revenue and the distribution of electoral costs associated with the inefficiency costs of taxation. Both effects may increase or reduce public redistribution depending on whether a change in the distribution of labor income increases or reduces the government's ability to collect tax revenue and the electoral costs of taxation. However, under general conditions, we show that if the covariance between labor income and the elasticity of the supply of labor and the labor income tax is negative (positive) then a change towards a dominant Lorenz curve for labor income leads to an increase (reduction) of the size of public redistribution.

We also show that changes in the distribution of remittances do not modify the size of public redistribution. This latter outcome is explained by the fact that for an economy with a tax on labor income, a change towards a dominant Lorenz curve of remittances does not affect both the electoral costs associated with taxation and the electoral benefits from public transfers. As a result, parties do not respond to a change in the distribution of income caused by remittances.

We extend our analysis to consider an income tax on all sources of private income (labor income and remittances) but exempts public transfers. In this case we find that changes in the distribution of labor income and remittances also matter to determine the size of public redistribution. To be specific, a fall in the inequality of labor income induces an increase in public redistribution due to a higher tax revenue collection and, simultaneously, a fall in public redistribution due to an increase in the parties' electoral costs from the inefficiencies on the households' supply of labor explained by the income tax. If the tax revenue effect dominates (is dominated by) the rise in the electoral costs from inefficient taxation then the size of public redistribution increases (falls).

However, a fall in the distribution of income caused by a reduction in the inequality of remittances induces an increase in public redistribution due to a higher tax revenue collection but it also produces an ambiguous effect on redistribution due to the electoral costs from the inefficiencies on the remittances-income tax elasticity might be higher if this elasticity is negative and lower if the elasticity is positive. As a result, if the remittances-income tax elasticity is positive then unambiguously a fall in the inequality of remittances induces the government to increase

redistribution. This is the case because a change towards a dominant Lorenz curve of remittances increases the government's tax revenue, which effect tends to increase redistribution, and reduces the electoral costs from inefficient taxation which effect also tends to increase the size of public redistribution.

However if the remittances-income tax elasticity is negative, the electoral costs from inefficient taxation increase which induces parties to reduce redistribution. In this case, the net change on public redistribution depends on whether the tax revenue effect, which tends to increase redistribution, dominates (is dominated by) the rise in the electoral costs from inefficient taxation which effect tends to reduce redistribution.

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Appendix

Lemma 1.

For an economy with a wage income tax system τ^i and a universal per capita transfer T^i , the politically optimal size of $T^i \forall i = 1,2$ is given by:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w)z(w)dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha z(w)dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}}{-\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w)dw} \right\} \int_{w_0}^y h(w)w\ell(\tau^i, w, R)dw \quad (\text{A.1})$$

Proof.

The problem of policy design for party i is:

$$\begin{aligned} \text{Max } \phi^i &= \int_{w_0}^y h(w) F^i(\chi^i(w)) dw \quad \forall i = 1,2 \\ \text{s. t: } T^i &= \tau^i \int_{w_0}^y h(w)w\ell(\tau^i, w, R)dw \quad \forall y \in [w_0, w_{max}] \end{aligned} \quad (\text{A.2})$$

Define the Lagrangian $\delta^i = \phi^i + \lambda^i [\tau^i \int_{w_0}^y h(w)w\ell(\tau^i, T^i, w, R)dw - T^i]$ where λ^i as a Lagrange multiplier for the parties' problem. The first order conditions of this problem are $\frac{\partial \delta^i}{\partial \tau^i} = 0 \quad \forall \tau^{*i} > 0$, $\frac{\partial \delta^i}{\partial T^i} = 0 \quad \forall T^{*i} > 0$ and $\frac{\partial \delta^i}{\partial \lambda^i} = 0 \quad \forall \lambda^{*i} \neq 0$ where:

$$\begin{aligned} \frac{\partial \phi^i}{\partial \tau^i} &= \int_{w_0}^y h(w) f^i(\chi^i(w)) \frac{\partial v}{\partial \tau^i} dw \\ + \lambda^i \int_{w_0}^y h(w) w \ell dw + \lambda^i \tau^{*i} \int_{w_0}^y h(w)w \frac{\partial \ell}{\partial \tau^i} dw &= 0 \end{aligned} \quad (\text{A.3})$$

and ²⁰

$$\frac{\partial \phi^i}{\partial T^i} = \int_{w_0}^y h(w) f^i(\chi^i(w)) \frac{\partial v}{\partial T^i} dw - \lambda^i = 0 \quad (\text{A.4})$$

Define $z(w) = w\ell(\tau^i, w, R)$ as the voter's labor income. The labor-income tax elasticity is $\varepsilon_{\ell-\tau^i} = \frac{\partial \ell}{\partial \tau^i} \frac{1}{\ell}$. Use in the first order conditions $\frac{\partial v}{\partial \tau^i} = -\alpha z(w)$ and $\frac{\partial v}{\partial T^i} = \alpha$ where α is the voter's marginal utility of income to re-write (A.3) as follows:

²⁰ Following the literature, see Myles (2002), we ignore the effect of public transfers on the household's supply of labor. This is a common practice in the literature and we do so for mathematical simplicity.

$$\tau^{*i} = \frac{\int_{w_0}^y h(w) z(w) dw - \frac{1}{\lambda^i} \int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha z(w) dw}{-\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w) dw} \quad (\text{A.5})$$

From (A.4) $\lambda^i = \int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw$. Use the budget constraint of the government to show:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w) z(w) dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha z(w) dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}}{-\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w) dw} \right\} \int_{w_0}^y h(w) z(w) dw \quad (\text{A.6})$$

Proposition 1.

Define:

1.1) $\sigma[f^i(\chi^i(w)) \alpha, z(w)]$ as the covariance between $f^i(\chi^i(w)) \alpha$ and $z(w)$,

1.2) $\mu(I(w)) = \int_{w_0}^{w_{\max}} h(w) I(w) dw$ as the average private income,

1.3) $L_{z-I}(p) = \int_{w_0}^y h(w) \Phi_{z-I} dw$ as the Lorenz curve of labor income

1.4) $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ as the covariance between the household's elasticity of the labor supply and income taxes, $\varepsilon_{\ell-\tau^i}$, and $z(w)$

1.5) $\mu(f^i(w)) = \int_{w_0}^{w_{\max}} h(w) f^i(\chi^i(w)) \alpha dw$ as the average marginal expected proportion of the votes that can be gained by providing a public transfer of \$1 to voters,

1.6) Define $\Phi_{f^i} = \frac{f^i(\chi^i(w)) \alpha}{\int_{w_0}^{w_{\max}} h(w) f^i(\chi^i(w)) \alpha dw}$: $L_{f^i}(p) = \int_{w_0}^y h(w) \Phi_{f^i} dw$ with $L_{f^i}(p)$ as a Lorenz curve of electoral influence

1.7) $\int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw$ as the economy's average elasticity of the labor supply and income taxes

Therefore, the universal per capita transfer $T^{*i} \forall i = 1,2$ can be expressed as follows:

$$\begin{aligned} T^{*i} &= \\ &= \frac{-\sigma[f^i(\chi^i(w)) \alpha, z(w)] \mu(I(w)) L_{z-I}(p)}{\{-\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w)) L_{z-I}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw\} \{\mu(f^i(w)) L_{f^i}(p)\}} \end{aligned} \quad (\text{A.7})$$

Proof.

Define $\sigma[f^i(\chi^i(w))\alpha, z(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and $z(w)$ and $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ as the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$ then the following is satisfied

$$\begin{aligned} \int_{w_0}^y h(w)f^i(\chi^i(w))\alpha z(w)dw &= \\ &= \sigma[f^i(\chi^i(w))\alpha, z(w)] \\ &\quad + \int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw \left\{ \int_{w_0}^y h(w)z(w)dw \right\} \end{aligned} \quad (A.8)$$

and

$$\begin{aligned} \int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}z(w)dw &= \\ &= \sigma[\varepsilon_{\ell-\tau^i}, z(w)] + \int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw \left\{ \int_{w_0}^y h(w)z(w)dw \right\} \end{aligned} \quad (A.9)$$

Define $I(w) = w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)$ as the private income of voters with wage w and Φ_{z-I} and Φ_{fi} as follows:

$$\Phi_{z-I} = \frac{z(w)}{\int_{w_0}^{w_{max}} h(w)I(w)dw} \quad (A.10)$$

$$\Phi_{fi} = \frac{f^i(\chi^i(w))\alpha}{\int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw} \quad (A.11)$$

Moreover consider that $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in (0,1)$ which satisfies.

$$p(y) = \int_{w_0}^y h(w)dw \quad (A.12)$$

Hence we define the Lorenz curve of labor income, $L_z(p)$, as follows

$$L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw \quad \forall y \in [w_0, w_{max}] \quad (A.13)$$

and the Lorenz curve for electoral influence is

$$L_{fi}(p) = \int_{w_0}^y h(w)\Phi_{fi}dw \quad \forall y \in [w_0, w_{max}] \quad (A.14)$$

Let $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ be the average private income and $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w) f^i(\chi^i(w)) \alpha dw$ be the average marginal expected proportion of votes that can be gained by providing a public transfer of \$1 to voters. We follow Lambert (2001) and define the generalized Lorenz curves of labor income and electoral influence, respectively, by $GL_z(p)$ and $GL_{f^i}(p)$ such that:

$$\begin{aligned} \exists p(y) = \int_{w_0}^y h(w)dw \implies GL_z(p) &= \int_{w_0}^y h(w)z(w)dw = \\ &= \mu(I(w))L_{z-I}(p) \end{aligned} \tag{A.15}$$

and

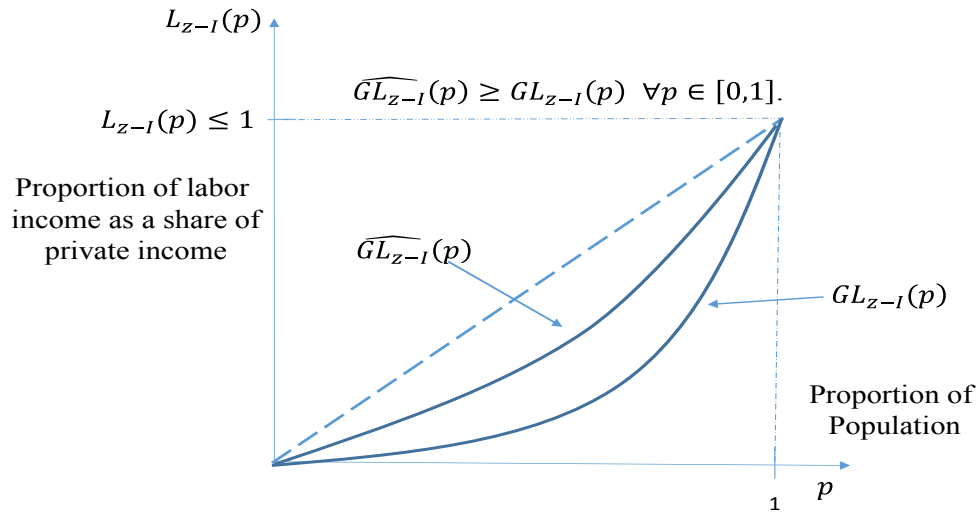
$$\begin{aligned} \exists p(y) = \int_{w_0}^y h(w)dw \implies GL_{f^i}(p) &= \int_{w_0}^y h(w)f^i(\chi^i(w)) \alpha dw = \\ &= \mu(f^i(w))L_{f^i}(p) \end{aligned} \tag{A.16}$$

Use (A.8) to (A.16) into (A.6) from Lemma 1 to obtain:

$$\begin{aligned} T^{*i} &= \\ &= \frac{-\sigma [f^i(\chi^i(w)) \alpha, z(w)] \mu(I(w))L_{z-I}(p)}{\{-\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - \{\mu(I(w))L_{z-I}(p)\} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw\} \{\mu(f^i(w))L_{f^i}(p)\}} \end{aligned} \tag{A.17}$$

Definition. Lorenz dominance. Assume two distributions $z(w)$ and $\hat{z}(w)$ leading to generalized Lorenz curves $GL_{z-I}(p) = \mu(I(w))L_{z-I}(p)$ with $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_z dw$ and $\widehat{GL}_{z-I}(p) = \mu(I(w))\widehat{L}_{z-I}(p)$ with $\widehat{L}_{z-I}(p) = \int_{w_0}^y h(w)\widehat{\Phi}_z dw \ \forall y \in [w_0, w_{max}]$ such that the distribution $\hat{z}(w)$ Lorenz-dominates $z(w)$ implying $\widehat{GL}_{z-I}(p) \geq GL_{z-I}(p) \ \forall p \in [0,1]$.

Figure A.2. Lorenz Labor Income Dominance for Constant Average Income and the share of Labor Income in Relation to Private Income



Proposition 3.

The size of remittances of households living abroad is given by $R(\tau^i, T^i, w^a, w) \in \operatorname{argmax} \mu^a(c^a, \ell^a, v(\tau, T, w, R))$ s. t: $c^a = w^a \ell^a - R$. Define $\varepsilon_{\ell-R} = \frac{\partial \ell}{\partial R} \frac{R}{\ell}$ and assume leisure is a normal good then $\varepsilon_{\ell-R} < 0$. Moreover,

$$\text{If } \left(\varepsilon_{\ell-R} \frac{w\ell}{R} + 1 \right) < 0 \text{ then } \frac{dR}{d\tau^i} > 0 \tag{A.18}$$

Proof

The optimal choices of nationals living abroad are:²¹

$$c^a(\tau^i, T^i), \ell^a(\tau^i, T^i), R(\tau^i, T^i) \in \operatorname{argmax} \left\{ \begin{array}{l} \mu^a(c^a, \ell^a, v(\tau^i, T^i, w, R)) \\ \text{s. t: } c^a = n^a \ell^a - R \end{array} \right\} \tag{A.19}$$

To obtain the first order conditions use

$$v(\tau, T, w, R) = \mu^* \left((w\ell(\tau^i, T^i, w, R) + R)(1 - \tau^i) + T^i, \ell(\tau^i, T^i, w, R) \right) \tag{A.20}$$

To state the following:

²¹ For constraints in the use of space we denote the household's optimal choices $c^a(\tau^i, T^i)$ as $c^a(\tau^i, T^i, w^a, w)$, $\ell^a(\tau^i, T^i)$ as $\ell^a(\tau^i, T^i, w^a, w)$ and $R(\tau^i, T^i)$ as $R(\tau^i, T^i, w^a, w)$ when we consider convenient.

$$\mu^a = \mu^a(w^a \ell^a - R, \ell^a, \mu^*(\{w\ell(\tau^i, T^i, w, R) + R\}(1 - \tau) + T, \ell(\tau^i, T^i, w, R))) \quad (\text{A. 21})$$

From the expression above find FOC's $d\mu^a/d\ell^a$ and $d\mu^a/dR$:

$$\frac{d\mu^a}{d\ell^a} = \frac{\partial \mu^a}{\partial c^a} w^a + \frac{\partial \mu^a}{\partial \ell^a} = 0 \quad \forall \ell^a(\tau^i, w^a, w) > 0 \quad (\text{A. 22})$$

and

$$\begin{aligned} \frac{d\mu^a}{dR} &= -\frac{\partial \mu^a}{\partial c^a} + \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \left(\frac{\partial \ell}{\partial R} w + 1 \right) (1 - \tau^i) + \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial \ell} \frac{\partial \ell}{\partial R} \\ &= 0 \quad \forall R(\tau^i, T^i, w^a, w) > 0 \end{aligned} \quad (\text{A. 23})$$

Totally differentiate the first order conditions to show:

$$\begin{bmatrix} \frac{\partial^2 \mu^a}{\partial^2 \ell^a} & \frac{\partial^2 \mu^a}{\partial \ell^a \partial R} \\ \frac{\partial^2 \mu^a}{\partial R \partial \ell^a} & \frac{\partial^2 \mu^a}{\partial^2 R} \end{bmatrix} \begin{bmatrix} d\ell^a \\ dR \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\partial^2 \mu^a}{\partial R \partial t} d\tau^i \end{bmatrix} \quad (\text{A. 24})$$

where:

$$-\frac{\partial^2 \mu^a}{\partial R \partial t} d\tau^i = \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \left(\frac{\partial \ell}{\partial R} w + 1 \right) = \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \left(\varepsilon_{\ell-R} \frac{w\ell}{R} + 1 \right) \text{ and where } \varepsilon_{\ell-R} = \frac{\partial \ell}{\partial R} \frac{R}{\ell}.$$

Define:

$$H_{\mu^a} = \begin{bmatrix} \frac{\partial^2 \mu^a}{\partial^2 \ell^a} & \frac{\partial^2 \mu^a}{\partial \ell^a \partial R} \\ \frac{\partial^2 \mu^a}{\partial R \partial \ell^a} & \frac{\partial^2 \mu^a}{\partial^2 R} \end{bmatrix} \quad (\text{A. 25})$$

By assumption, $\mu^a(c^a, \ell^a, v(\tau^i, T^i, w, R))$ is a strict quasi-concave preference relation which implies that H_{μ^a} is a negative definite matrix therefore:

$$\partial^2 \mu^a / \partial^2 \ell^a < 0$$

and

$$|H_{\mu^a}| = \{\partial^2 \mu^a / \partial^2 \ell^a\} \{\partial^2 \mu^a / \partial^2 R\} - \{\partial^2 \mu^a / \partial \ell^a \partial R\}^2 > 0.$$

It follows that the best response of remittances to taxation is given by

$$\begin{aligned} \frac{dR}{d\tau^i} &= \frac{\begin{bmatrix} \frac{\partial^2 \mu^a}{\partial^2 \rho^a} & 0 \\ \frac{\partial^2 \mu^a}{\partial R \partial \rho^a} & \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \left(\varepsilon_{\ell-R} \frac{w\ell}{R} + 1 \right) \end{bmatrix}}{|H_{\mu^a}|} = \\ &= \frac{1}{|H_{\mu^a}|} \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \left(\varepsilon_{\ell-R} \frac{w\ell}{R} + 1 \right) \left\{ \frac{\partial^2 \mu^a}{\partial^2 \rho^a} \right\} \begin{matrix} \geq 0 \\ < 0 \end{matrix} \end{aligned} \quad (A.26)$$

Because

$$|H_{\mu^a}| > 0, \frac{\partial \mu^a}{\partial \mu^*} \frac{\partial \mu^*}{\partial c} \geq 0 \text{ and } \frac{\partial^2 \mu^a}{\partial^2 \rho^a} < 0, \text{ if } \left(\varepsilon_{\ell-R} \frac{w\ell}{R} + 1 \right) \begin{matrix} \leq 0 \\ > 0 \end{matrix} \text{ then } \frac{dR}{d\tau^i} \begin{matrix} \geq 0 \\ < 0 \end{matrix}. \quad \blacksquare$$

Lemma 2. Assume a tax system with a personal income tax τ^{*i} on labor income and interfamily private transfers. For this economy $T^{*i} \forall i = 1,2$ is given by:

$$\begin{aligned} T^{*i} &= \\ &= \left\{ \begin{array}{l} \int_{w_0}^y h(w) I(w) dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w) dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw} \\ - \left\{ \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w) dw + \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} R dw \right\} \end{array} \right\} \int_{w_0}^y h(w) I(w) dw \quad (A.27) \end{aligned}$$

Proof.

For this case, the parties' problem is

$$\begin{aligned} \text{Max } \phi^i &= \int_{w_0}^y h(w) F^i(\chi^i(w)) dw \quad \forall i = 1,2 \\ \text{s. t. } T^i &= \tau^i \int_{w_0}^y h(w) \{w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)\} dw \\ \forall y &\in [w_0, w_{max}] \end{aligned} \quad (A.28)$$

Define the Lagrangian:

$$\delta^i = \phi^i + \lambda^i \left[\tau^i \int_{w_0}^y h(w) \{w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)\} dw - T^i \right]$$

where λ^i as a Lagrange multiplier for the parties' problem. The first order conditions of this problem are $\frac{\partial \delta^i}{\partial \tau^i} = 0 \forall \tau^{*i} > 0$, $\frac{\partial \delta^i}{\partial T^i} = 0 \forall T^{*i} > 0$ and $\frac{\partial \delta^i}{\partial \lambda^i} = 0 \forall \lambda^{*i} \neq 0$, where:

$$\begin{aligned} \frac{\partial \delta^i}{\partial \tau^i} &= \int_{w_0}^y h(w) f^i(\chi^i(w)) \frac{\partial v}{\partial \tau^i} dw \\ &+ \lambda^i \int_{w_0}^y h(w) \{w\ell + R\} dw + \lambda^i \tau^{*i} \int_{w_0}^y h(w) \left\{ w \frac{\partial \ell}{\partial \tau^i} + \frac{\partial R}{\partial \tau^i} \right\} dw = 0 \end{aligned} \quad (\text{A.29})$$

and

$$\frac{\partial \delta^i}{\partial T^i} = \int_{w_0}^y h(w) f^i(\chi^i(w)) \frac{\partial v}{\partial T^i} dw - \lambda^i = 0 \quad (\text{A.30})$$

Moreover, $\frac{\partial \delta^i}{\partial \lambda^i} = 0 \forall \lambda^{*i} \neq 0$ implies $T^{*i} = \tau^{*i} \int_{w_0}^y h(w) \{w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)\} dw \quad \forall y \in [w_0, w_{max}]$. Define $I(w)$ as the private income of voters with wage w :

$$I(w) = w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w) \quad (\text{A.31})$$

The labor-income tax elasticity is $\varepsilon_{\ell-\tau^i} = \frac{\partial \ell}{\partial \tau^i} \frac{1}{\ell}$ and the remittances-income tax elasticity is $\varepsilon_{R-\tau^i} = \frac{\partial R}{\partial \tau^i} \frac{1}{R}$. Use in the first order conditions $\frac{\partial v}{\partial \tau^i} = -\alpha I(w)$ and $\frac{\partial v}{\partial T^i} = \alpha$ where α is the voter's marginal utility of income to state (A.31) as follows:

$$\tau^{*i} = \frac{\int_{w_0}^y h(w) \{w\ell + R\} dw - \frac{1}{\lambda^i} \int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha \{w\ell + R\} dw}{-\left\{ \int_{w_0}^{w_{max}} h(w) \varepsilon_{\ell-\tau^i} z(w) dw + \int_{w_0}^{w_{max}} h(w) \varepsilon_{R-\tau^i} R dw \right\}} \quad (\text{A.32})$$

Use $\lambda^i = \int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw$ from (A.30) and the government's budget constraint to show:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w) I(w) dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w) dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}}{-\left\{ \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} z(w) dw + \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} R dw \right\}} \right\} \int_{w_0}^y h(w) I(w) dw \quad (\text{A.33})$$

Proposition 4.

Define:

4.1) $\sigma [f^i(\chi^i(w))\alpha, I(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and the household's private income $I(w) = w\ell(\tau^i, w) + R(\tau^i, T^i, w^a, w)$;

4.2) $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ as the average private income;

4.3) $L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z-I}dw$ as the Lorenz curve of labor income;

4.4) $L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw$ as the Lorenz curve of remittances;

4.5) $\Psi_{w\ell-\tau^i} = -\sigma[\varepsilon_{\ell-\tau^i}, z(w)] - s_{z-I}\{\mu(z(w))L_{z-I}(p)\} \int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw$

where $\sigma[\varepsilon_{\ell-\tau^i}, z(w)]$ is the covariance between $\varepsilon_{\ell-\tau^i}$ and $z(w)$, $s_{z-I} = \frac{\mu(z(w))}{\mu(I(w))}$ is the share of average labor income with respect the average of private income, $\mu(z(w)) = \int_{w_0}^{w_{max}} h(w)z(w)dw$ is the average labor income, and $\int_{w_0}^y h(w)\varepsilon_{\ell-\tau^i}dw$ is the economy's aggregate elasticity of labor supply and income taxation;

4.6) $\Psi_{R-\tau^i} = -\sigma[\varepsilon_{R-\tau^i}, R(w)] - s_{R-I}\{\mu(R(w))L_{R-I}(p)\} \int_{w_0}^y h(w)\varepsilon_{R-\tau^i}dw$

where $\sigma[\varepsilon_{R-\tau^i}, R(w)]$ is the covariance between $\varepsilon_{R-\tau^i}$ and R , $s_{R-I} = \frac{\mu(R(w))}{\mu(I(w))}$ is the share of average interfamily private transfers with respect the average of private income, $\mu(R(w)) = \int_{w_0}^{w_{max}} h(w)Rdw$ is the average size of remittances, and $\int_{w_0}^y h(w)\varepsilon_{R-\tau^i}dw$ is the economy's aggregate elasticity of interfamily private transfers and income taxation;

4.7) $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$ as the average marginal expected proportion of the votes that can be gained by providing a public transfer of \$1 to voters.

4.8) Define $\Phi_{f^i} = \frac{f^i(\chi^i(w))\alpha}{\int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw}$: $L_{f^i}(p) = \int_{w_0}^y h(w)\Phi_{f^i}dw$ with $L_{f^i}(p)$ as a Lorenz curve of electoral influence.

Therefore, the universal per capita transfer $T^{*i} \forall i = 1,2$ can be expressed as follows:

$$T^{*i} = \left\{ \frac{-\sigma [f^i(\chi^i(w))\alpha, I(w)]\mu(I(w))\{L_{z-I}(p) + L_{R-I}(p)\}}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\}\{\mu(f^i(w))L_{f^i}(p)\}} \right\} \quad (A.34)$$

Proof.

Consider the equilibrium transfer characterized in lemma 2:

$$T^{*i} = \left\{ \frac{\int_{w_0}^y h(w)I(w)dw - \frac{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w)dw}{\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw}}{-\left\{ \int_{w_0}^y h(w) \varepsilon_{\rho-\tau^i} z(w)dw + \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} Rdw \right\}} \right\} \int_{w_0}^y h(w)I(w)dw \quad (\text{A.35})$$

Define $\sigma[f^i(\chi^i(w))\alpha, I(w)]$ as the covariance between $f^i(\chi^i(w))\alpha$ and $I(w)$, $\sigma[\varepsilon_{\rho-\tau^i}, z(w)]$ as the covariance between $\varepsilon_{\rho-\tau^i}$ and $z(w)$, and $\sigma[\varepsilon_{R-\tau^i}, R]$ as the covariance between $\varepsilon_{R-\tau^i}$ and R then the following is satisfied:

$$\int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha I(w) dw = \sigma[f^i(\chi^i(w))\alpha, I(w)] + \int_{w_0}^y h(w) f^i(\chi^i(w)) \alpha dw \int_{w_0}^y h(w) I(w) dw \quad (\text{A.36})$$

and

$$\int_{w_0}^y h(w) \varepsilon_{\rho-\tau^i} z(w) dw = \sigma[\varepsilon_{\rho-\tau^i}, z(w)] + \int_{w_0}^y h(w) \varepsilon_{\rho-\tau^i} dw \int_{w_0}^y h(w) z(w) dw \quad (\text{A.37})$$

and

$$\int_{w_0}^y h(w) \varepsilon_{R-\tau^i} R dw = \sigma[\varepsilon_{R-\tau^i}, R(w)] + \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw \int_{w_0}^y h(w) R(w) dw \quad (\text{A.38})$$

Define, $\Phi_{z(w)-I}$, Φ_{R-I} and Φ_{f^i} as follows:

$$\Phi_{z(w)-I} = \frac{z(w)}{\int_{w_0}^{w_{max}} h(w)I(w)dw} \quad (\text{A.39})$$

$$\Phi_{R-I} = \frac{R}{\int_{w_0}^{w_{max}} h(w)I(w)dw} \quad (\text{A.40})$$

$$\Phi_{f^i} = \frac{f^i(\chi^i(w))\alpha}{\int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw} \quad (\text{A.41})$$

Moreover consider that $\forall y \in [w_0, w_{max}] \exists$ a rank $p \in (0,1)$ which satisfies.

$$p(y) = \int_{w_0}^y h(w)dw \quad (A.42)$$

Hence, we define the Lorenz curve of labor income-total private income, $L_{z-I}(p)$, as follows:

$$L_{z-I}(p) = \int_{w_0}^y h(w)\Phi_{z(w)-I}dw \quad \forall y \in [w_0, w_{max}] \quad (A.43)$$

The Lorenz curve for remittances-total private income $L_{R-I}(p)$ is:

$$L_{R-I}(p) = \int_{w_0}^y h(w)\Phi_{R-I}dw \quad \forall y \in [w_0, w_{max}] \quad (A.44)$$

and the Lorenz curve for electoral influence is

$$L_{f^i}(p) = \int_{w_0}^y h(w)\Phi_{f^i}dw \quad \forall y \in [w_0, w_{max}] \quad (A.45)$$

Let $\mu(z(w)) = \int_{w_0}^{w_{max}} h(w)z(w)dw$ be the average labor income, $\mu(R(w)) = \int_{w_0}^{w_{max}} h(w)R(w)dw$ be the average size of remittances, $\mu(I(w)) = \int_{w_0}^{w_{max}} h(w)I(w)dw$ be the average private income, and $\mu(f^i(w)) = \int_{w_0}^{w_{max}} h(w)f^i(\chi^i(w))\alpha dw$ be the average marginal expected proportion of the votes that can be gained (lost) by providing (or taking away) \$1 to voters.

As before, we define the generalized Lorenz curves of labor income-total private income, remittances-total private income and electoral influence, respectively, by $GL_{z-I}(p)$, $GL_{R-I}(p)$ and $GL_{f^i}(p)$:

$$p(y) = \int_{w_0}^y h(w)dw \implies GL_{z-I}(p) = \int_{w_0}^y h(w)z(w)dw = \mu(I(w))L_{z-I}(p) \quad (A.46)$$

$$p(y) = \int_{w_0}^y h(w)dw \implies GL_{R-I}(p) = \int_{w_0}^y h(w)R(w)dw = \mu(I(w))L_{R-I}(p) \quad (A.47)$$

and

$$\begin{aligned} p(y) = \int_{w_0}^y h(w)dw \implies GL_{f^i}(p) &= \int_{w_0}^y h(w)f^i(\chi^i(w))\alpha dw = \\ &= \mu(f^i(w))L_{f^i}(p) \end{aligned} \quad (A.48)$$

Use (A.36) to (A.48) into (A.35) to show:

$$T^{*i} = \left\{ \frac{-\sigma \left[f^i \left(\chi^i(w) \right) \alpha, I(w) \right] \mu(I(w)) \{L_{Z-I}(p) + L_{R-I}(p)\}}{\{\Psi_{w\ell-\tau^i} + \Psi_{R-\tau^i}\} \{\mu(f^i(w))L_{f^i}(p)\}} \right\} \quad (A.49)$$

Where:

$$\Psi_{w\ell-\tau^i} = -\sigma \left[\varepsilon_{\ell-\tau^i}, z(w) \right] - s_{Z-I} \{ \mu(I(w))L_{Z-I}(p) \} \int_{w_0}^y h(w) \varepsilon_{\ell-\tau^i} dw \quad (A.50)$$

$$\Psi_{R-\tau^i} = -\sigma \left[\varepsilon_{R-\tau^i}, R(w) \right] - s_{R-I} \{ \mu(I(w))L_{R-I}(p) \} \int_{w_0}^y h(w) \varepsilon_{R-\tau^i} dw \quad (A.51)$$

$$s_{Z-I} = \frac{\mu(z(w))}{\mu(I(w))} \quad \text{and} \quad s_{R-I} = \frac{\mu(R(w))}{\mu(I(w))} \quad (A.52)$$