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Multi-dimensional Skills and Matching: Implications for International Trade and Wage Inequality

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Abstract: Workers have various kinds of skills and abilities in different amounts and proportions. The technology of firms in an industry is also characterized by a certain skill combination. The combinations of skills supplied by workers are often not the same as those demanded by firms---there can be mismatches between the skills supplied by workers and those demanded by firms. This kind of mismatches can cause both inter- and intra-industry wage inequalities. By using a two-country two-industry model, I show that trade liberalization induces some workers in the importing industry to move to the exporting industry that offers a higher wage income than the importing industry after liberalization and, at the same time, some of the moving workers who are matched with less appropriate firms in the exporting industry result in a lower wage income than in autarky.

JEL classification: E24, F16, J24
Keywords: Skill mismatch, International trade, Wage inequality

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1 Introduction

The skills and abilities of people are often measured by their education level (e.g., high school, university, graduate school, and so on). Reflecting this method of measurement, the skills and abilities of a person are expressed by a one-dimensional variable in economic analysis. In the labor market, a high-skilled person can obtain a job with a high salary, while a low-skilled person often ends up with a low salary. However, there are various types of skills that are not comparable with each other. For example, a salesperson should possess good communication skills, an information technology specialist should have precise computational skills, and an artist should have delicate aesthetic senses and creative skills. Which person is the most "skilled," a person with good communication skills, a person with precise computational skills, or a person with delicate aesthetic senses? Of course, we cannot answer such a (meaningless) question, because those skills are not comparable. Hence, I cannot determine which of these individuals has the best skills and abilities if they are judged using only one dimension. Individuals possess various skills in different amounts and proportions. Therefore, a person's skill is not measured by just one dimension but by multiple dimensions. Each worker possesses a particular combination of skills and abilities, but the bundle or combination of skills is different from person to person.

This article considers two skills or abilities: a computational skill and a communication skill. The skill bundle that an individual possesses is assumed to be inseparable and non-additive. For example, an individual may have a better communication skill but weaker computational skills than another. I consider two industries, and that technology of firms in an industry is characterized by a certain skill proportion; to simplify the exposition, I assume that each industry has a Leontief technology. From the assumption, the character of a firm is described by the ratio of the two skills. Each firm provides any workers the opportunity to exert the fixed ratio of abilities. Workers use their two skills for production at the level required by firms. Since the skill bundles of workers are heterogeneous, there are few workers who have the skill bundle required by firms. Therefore, all workers cannot exert their skills at maximum. The gap between the skill demanded by firm and those supplied by workers gets lower the worker's wage income. I refer to the skill gap on production as a skill mismatch between firms and workers. When the skill mismatch occurs, some workers cannot maximize their abilities, eventually resulting in intra-industry wage inequality.

The importance of multi-skill problem has been recognized by some empirical literature. Guvenen et al. (2018) and Lise and Postel-Vinay (2016), by using US data, have estimated the skill matching between occupation and workers, and have discovered the mismatch in some types of skill. Especially, Guvenen et al. (2018) showed that the skill mismatch lowers current and future wages. Although their analysis is not in a context of international trade, the latter may play an important role for skill mismatch by changing workers' occupation choice. In my study, I analyze the matching patterns between firms and workers, industry structures, and the intra- and inter-industry wage inequalities. The distribution of skills is also different from country to country. This difference in the skill distributions can cause the differences in the industry structures of the countries. Consequently, cross-country differences in industry structure can be a source of comparative advantages. I consider international trade between two symmetric countries that share the same technology, preferences, and size of labor; however, both countries have different distribution of skills. When international trade is allowed, a portion of the workers employed in an importing industry will move to an exporting industry since they can earn a higher wage income than the wage income they receive if they remain in the former. However, the moving workers who are matched with less appropriate firms may suffer from a lower wage income than in autarky.

This paper relates to the literature on skill matching and that on the market imperfections and international trade. Roy (1950) and Rosen (1978) have shown that multiple dimensions of skills affect the wage distribution and the industry structure.

However, they have not examined the implications on international trade. Grossman and Maggi (2000), Bougheas and Ritzman (2007), Costinot and Vogel (2010), and Burnstein et al. (2015) have connected Roy's model with international trade and investigated the relationship between the distribution of skills and comparative advantages and the effects of globalization on the wage inequality. However, they have treated one-dimensional skill. Grinrols and Matusz (1988) have analyzed the relationship between multiple skills and international trade, but they focus on welfare analysis and not wage inequality.

The work of Ohnsorge and Trefler (2007) is the first to analyze the relationship among two-dimensional skill sets, international trade, and income inequality. Since this paper is quite related to Onsroge and Trefler (2007), I refer to Ohnsorge and Trefler (2007) as O&T. They have shown that the two-dimensional heterogeneity of workers' skills affects job choice, and that cross-country differences in the distribution of skills determine the patterns of international trade. O&T considered two types of skills, communication and quantitative skills, and that the skill bundle is heterogeneous among workers. They considered that there is a continuum of industry, and that each industry has different demands for skill bundles. Workers seek jobs and select the one offering the highest wage.

O&T showed that the comparative advantage of the two skills determines the pattern of worker sorting. When a worker has relatively stronger quantitative than communication skills, he or she engages in a more quantitative-intensive industry. Adao (2015) and Galle et al. (2017) used multi-sector and multi-skill model and analyzed the quantitative effects of international trade on wage inequality empirically. This model also deals with two-dimensional skills and has a high affinity to my model. However, neither O&T, nor Adao (2015), nor Galle et al. (2017) considered the possibility of mismatches of skill demand and supply between firms and workers. Since the previous researches consider the continuum of industry, all workers can select the job that is more appropriate for them. I intend to introduce the mismatch problem of the demand and supply of skill combinations in production function, and analyze how the demand-supply mismatches affect the industry output, wage inequality, and international trade.

Second, there is an extensive amount of researches concerning the relationship between international trade and the labor market. Traditional trade theories have paid less attention to the roles of the labor market in international trade. However, recently, a number of studies focusing on this issue have arisen. These recent studies have shown that the problems of labor markets affect output, wage inequality, and international trade. Some researchers have focused on labor market friction in job seeking (e.g., Helpman et al. 2010a, 2010b and Ferlbermayr et al. 2011) on one hand, while some others on the concept of fairness in the labor market (e.g., Egger and Kreickemeier 2009, 2012). These papers considered that the mismatches between firms and workers are attributable to some search frictions or institutional arrangements in the labor market. This type of mismatch is different from the one in my paper. Furthermore, the skills analyzed in these papers are one-dimensional. Unlike these previous researches, my study introduces two-dimensional skills into the model and analyzes the relationship between the functioning of labor market and international trade.

The remainder of the paper is organized as follows. Section 2 explains the base of the model and the distribution of skills. Section 3 analyzes the relationship between the distribution of skills and international trade. Section 4 concludes with a summary and a discussion on future research.

2 The Model

2.1 Production

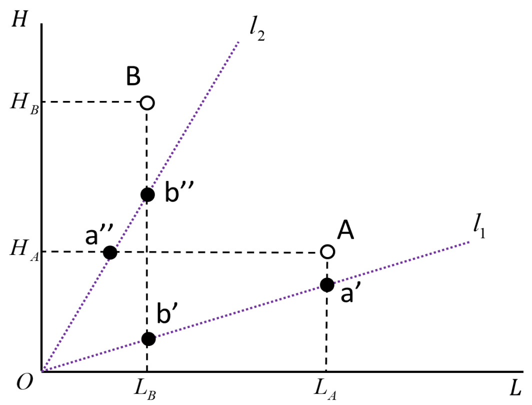
I consider an economy with two countries; in each country, two goods can be produced using only one factor of production (i.e., labor). Both goods and labor markets are perfectly competitive. An individual worker has two abilities, H and L, which represent the computational ability and the communication or teamwork skills, respectively. H and L are heterogeneously distributed among the workers. The skill bundle that an individual possesses is assumed to be inseparable and non-additive. Suppose that Industry 1 intensively uses L and industry 2 intensively does H. Technology of industry i $(i = 1,2)$ is described by the following Leontief function:

$$y_i = \min\left\{\frac{H}{\beta_i}, \frac{L}{1-\beta_i}\right\}, \quad i = 1,2 \#(1)$$

where y_i denotes the quantity of good i produce. I assume $\beta_1 < \beta_2$ since industry 2 more intensively uses quantitative skill H than industry 1. O&T only assumes constant-returns-to-scale technology and does not specify a production function. However, it is important to assume the Leontief technology for considering the mismatch described in this article. If you use the other function (i.e. the Cobb-Doulas function), it is not possible to reach my result. A firm cannot unbundle a worker's abilities, and considers only y_i important, and pays wages to a worker based on his/her achievement. From the cost minimization of a firm, we can derive the conditional demand for the abilities: $H = \beta_i y_i$and $L = (1 - \beta_i) y_i, i = 1,2$. Due to perfect competition, the wage for a worker is $W_i = P_i y_i$, where P_i is the price of good i.

Figure 1 plots the relationship between an individual skill set and production pattern. The quantitative ability H is measured vertically and the communication ability L is measured horizontally. Workers A and B, for example, have the skill bundles described by points A and B. Half line $l_i, i = 1,2$ denotes the locus of production implicated by the Leontief production function.

Figure 1. Relationship between Individual Skill Set and Production Pattern



Let us consider worker A. Worker A has a production possibility set, which is represented by rectangular area OH_AAL_A, and produces good 1 in point a' and good 2 in point a''. In point a', worker A can achieve his/her communication skill L_A at the maximum, but cannot achieve computational skill H_A in industry 1. The amount Aa' of computational skill (H) gets waste on production. In point a'', worker A can also achieve quantitative skill (H_A) at the maximum, but cannot achieve communication skill (L_A) in industry 2. The amount Aa'' of communication skill (L) gets waste on production. On the other hand, worker B has a production possibility set, which is represented by rectangular area OH_BBL_B, and the worker produces the good on H_BBL_B. From the assumption of the Leontief production function, worker B produces good 1 in point b' and good 2 in point b''. Worker B can achieve his/her communication skill L_B

at the maximum, but cannot achieve quantitative skill H_B in both industries. How much workers can exert their skill and workers' production patterns depend on how a worker's skill set (H, L) is close to the Leontief locus $l_i, i = 1,2$. Since the Leontief locus requires the specific combination of two skills, it is important to focus on the relative amount of a worker's skill H/L, rather than absolute amount of each skill, H and L.

2.1 Distribution of Workers

To make the analysis simple, I use logarithms of the variables and define some new variables:

$$\begin{aligned} l &\equiv \ln L \\ h &\equiv \ln H \\ s &\equiv \ln \frac{H}{L} \quad (2) \\ p_i &\equiv \ln P_i \\ \omega_i &\equiv \ln W_i = p_i + \ln y_i \end{aligned}$$

Further, I assume that s and l are subject to the bivariate normal distribution, that is, $F_{sl}(s, l)$.

$$\begin{bmatrix} s \\ l \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_l \\ \rho\sigma_s\sigma_l & \sigma_l^2 \end{bmatrix}\right) \quad (3)$$

where $|\rho| < 1$ is the correlation coefficient between s and l; μ is the mean of s; and $\sigma_j, j = 1,2$ is the variance of s and l. O&T have empirically shown that $\rho < 0$ in advanced countries. We will discuss the meanings of negative ρ later in Section 3. To obtain some clear-cut results, we sometimes assume $\sigma_s = \sigma_l = 1$.

2.2 Sorting of Workers

I assume that there is a mass of workers and set the scale of workers to 1. All workers must decide the industry in which they work. From the discussion in the section 2.1, firms in each industry use a certain skill ratio, H/L, which is described by the Leontief locus $l_i, i = 1,2$ in Figure 1. I refer to the ratio as the firm's skill combination, which is solved by $\frac{H}{\beta_i} = \frac{L}{1-\beta_i}, i = 1,2$. From Eq. (2), I obtain the firm's skill combination of abilities in industry i:

$$s_i^* \equiv \ln \frac{H}{L}\Big|_{\frac{H}{\beta_i} = \frac{L}{(1-\beta_i)}} = \ln \beta_i - \ln(1 - \beta_i), i = 1,2 \quad (4)$$

Since the best skill combination is expressed by the ratio of H to L, the workers' skill combinations can be measured by $s = \ln H/L$. Worker with skill combination $s < s_1^*$ can produce the amount $s + l - \ln \beta_1$ of good 1 if he or she is employed in Industry 1, but the amount $s + l - \ln \beta_2$ if he or she is employed in Industry 2. On the other hand, workers with $s_1^* < s$, as worker A in Figure 1, produces $l - \ln(1-\beta_1)$ if employed in Industry 1 and $l - \ln(1-\beta_2)$ if employed in Industry 2. Workers with $s > s_2^*$ produces $l - \ln(1-\beta_1)$ if employed in Industry 1 and $l - \ln(1-\beta_2)$ if employed in Industry 2. Workers select the industries to be employed in based on the highest wage income. Since the production patterns depend on the ratio of two skills, s, I can consider the expected wage ω_i conditional on s:

$$\begin{aligned} \mathrm{E}[\omega_1(s,l)|s] &= \begin{cases} p_1 + s + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s-\mu) - \ln \beta_1, \text{if } s < s_1^* \\ p_1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s-\mu) - \ln(1-\beta_1), \text{if } s_1^* < s \end{cases} \\ \mathrm{E}[\omega_2(s,l)|s] &= \begin{cases} p_2 + s + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s-\mu) - \ln \beta_2, \text{if } s < s_2^* \\ p_2 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s-\mu) - \ln(1-\beta_2), \text{if } s_2^* < s \end{cases} \end{aligned} \#(5)$$

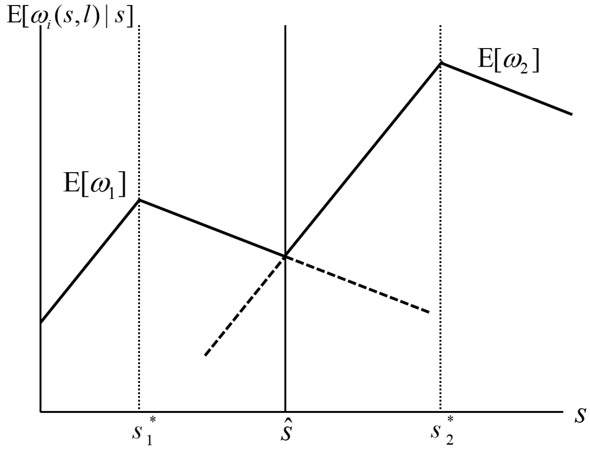
Differentiating the above expressions with respect to s, I obtain

$$\begin{aligned} \frac{\partial \mathrm{E}[\omega_1|s]}{\partial s} &= \begin{cases} 1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) > 0, \text{if } s < s_1^* \\ \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) < 0, \ \text{if } s_1^* < s \end{cases} \\ \frac{\partial \mathrm{E}[\omega_2|s]}{\partial s} &= \begin{cases} 1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) > 0, \text{if } s < s_2^* \\ \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) < 0, \text{if } s_2^* < s \end{cases} \end{aligned} \#(6)$$

where the signs are obtained when I assume $\sigma_s = \sigma_l = 1$ and $\rho < 0$. Figure 2 illustrates the relationship between the expected wage and s. From the nature of Eq. (6), the slope of ω_1 is positive when $s < s_1^*$ and negative when $s_1^* < s$. Similarly, the slope of ω_2 is positive when $s < s_2^*$ while it becomes negative when $s_2^* < s$. The gap from the firm's skill combination s_i^* $(i = 1,2)$ decreases the worker's expected wage income. Workers choose the industry where they earn the higher expected wage income. If a worker with skill combinations' earns more in Industry 1 than in Industry 2, $\mathrm{E}[\omega_1(s')] > \mathrm{E}[\omega_2(s')]$ he or she chooses to work in the former. In equilibrium, there must be a threshold $\hat{s}$ at which the worker is indifferent between working in Industry 1 or 2.

$$\hat{s} \equiv \ln\frac{H}{L}|_{\mathrm{E}[\omega_1]=\mathrm{E}[\omega_2]} = \ln\frac{\beta_2}{1-\beta_2}\frac{P_1}{P_2} \#(7)$$

Figure 2. Workers' Occupation Choice



A worker with $s < \hat{s}$ works in Industry 1 and one with $s < \hat{s}$ in Industry 2. Note that the level $\hat{s}$ is located between s_1^* and s_2^* whenever both goods are produced. From Eq. (6), $\hat{s}$ depends on the relative price of goods, P_1/P_2. I can confirm that the positive production of both industries requires $s_1^* < \hat{s} < s_2^*$, and I consider this case throughout the paper. Even if individuals work in the same industries, there exists the intra-industry income inequality. In Industry 1, for example, the expected wage in the skill combination range of $s < s_1^*$ increases with s while it decreases in the range of $s_1^* < s < \hat{s}$. Similarly, in Industry 2, as s increases, the wage that the labor with $\hat{s} < s < s_2^*$ earns increases, while it decreases with the group of $s_2^* < s$. This type of intra-industry income inequality stems not only from the absolute skill level, h and l, but also from the skill mismatches between firms and workers.

Sorting labor into industries, total output for both industries is determined. I define and calculate industry output of industry 1 and 2 in Appendix 1.

3 The Roles of Distribution and International Trade

I assume that there are two countries: Home and Foreign. Preferences, production technologies, and factor endowments (H and L) are the same between countries. There are no barriers to trade, therefore, consumers in both countries face the same prices $P_i^w, i = 1,2$. The differences between Home and Foreign are the distribution parameters: ρ, μ, and $\sigma_j, j = s, l$. These differences affect the production or industry structure of both countries, and become a source of comparative advantage. Section 3.1 analyzes the effects of ρ on the industry structure and the wages. Section

3.2 examines the effect of μ, and Section 3.3 focuses on the effect of $\sigma_j, j = s, l$.

3.1 Industry Structure and ρ

Parameter ρ is the correlation between s and l, and also expresses the correlation between the comparative and the absolute advantages. The variable $s \equiv \ln(H/L)$ represents the comparative skill advantage a worker has for the H-intensive industry (Industry 2) toward the L-intensive industry (Industry 1). For a given s, a large l means that the worker has an absolute skill advantage for both industries. O&T have empirically shown $\rho < 0$ in advanced countries. When all variables except ρ are the same in the two countries, the cross-country difference in ρ generates differences in the production structure. Therefore, the difference in ρ can be a cause of international trade. O&T showed that a country with a large ρ has a comparative advantage on the H-intensive industry in the case of the two industries. I can derive the same result as O&T model; the proof is given in Appendix 2.

Since the gap of the expected wage expands between high s and low s, the within-industry wage inequality gets worse in $s < s_1^*$ and $\hat{s} < s < s_2^*$ by the increase in ρ. On the other hand, the wage inequality in $s_1^* < s < \hat{s}$ and $s_2^* < s$ contracts. An increase in ρ reduces the slope of $\mathrm{E}[\omega_i(s,l)|s]$ among these groups and then narrows the wage gap between high s and low s.

In this section, I consider two different scenarios of international trade: free trade and gradual trade liberalization. First, I analyze the situation of free trade where two countries (Home and Foreign) begin free trade form autarky. Second, I assume a small country and examine a gradual reduction of trade tariff.

Free Trade

Suppose that Home has higher ρ than Foreign. Since preferences and production technologies are the same in both countries, free trade induces Home to export good 2 to Foreign; conversely, Foreign to export good 1 to Home. The difference in ρ also affects the wage inequality. First, let us consider a situation where ρ changes with the relative price being kept constant. Figure 3 describes the relationship between ρ and the expected wage conditioned on s. The dashed lines represent the expected wage after ρ increases. In $s < s_1^*$ and $\hat{s} < s < s_2^*$, the slope of $E[\omega_i(s,l)], i = 1,2$ becomes steeper as ρ increases.

International trade between Home and Foreign affects the within-industry wage inequality. Suppose that the autarkic price in Home and Foreign are p^h and p^f. Since Home has higher correlation between s and l, I obtain $p^h > p^f$. When international trade commences between them, the world relative price is determined between Home and Foreign;

$p^h > p^w > p^f$. The world relative price affects the threshold level of workers' occupational choice. Free trade give rise to the same threshold $\hat{s}^W$ in both countries. I obtain $\hat{s}^f < \hat{s}^w < \hat{s}^h$ where $\hat{s}^k, k = h, f, w,$ is the equilibrium rate of H and L at Home, the Foreign and World.

Figure 3. The Expected Wage Conditioned on s and ρ

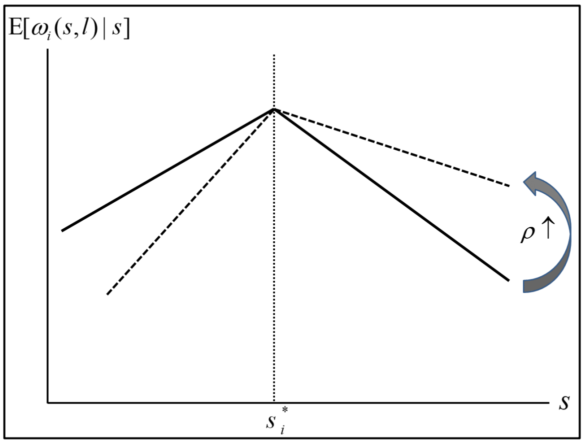
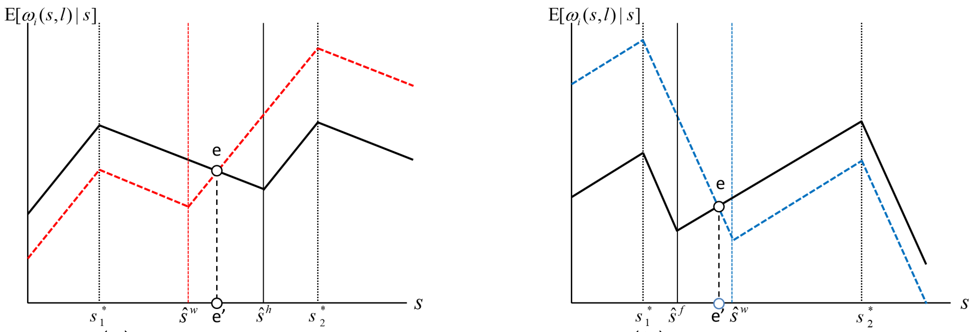


Figure 4 describes the expected wage profiles in Home and Foreign. We assume that Home has $\rho = -0.5$ and Foreign has $\rho^f < \rho^h$. The solid and dashed lines represent the situation in autarky and that under free trade, respectively. In Home, as international trade decreases relative prices, the number of workers who work in Industry 1 decreases and that in Industry 2 increases.

Figure 4. Wage and Profile in the Home Country



The change in the relative price also increases the wage income in Industry 2 but decreases that in Industry 1. The effects on wages mirror the Stolper-Samuelson theorem, which states that free trade raises the reward rates of factors used intensively in the exporting sector and reduces that of factors used intensively in the importing sector. Similarly, in Foreign, the wage income employed in Industry 2 increases while that employed in Industry 1 decreases. Free trade expands the cross-industry income inequality.

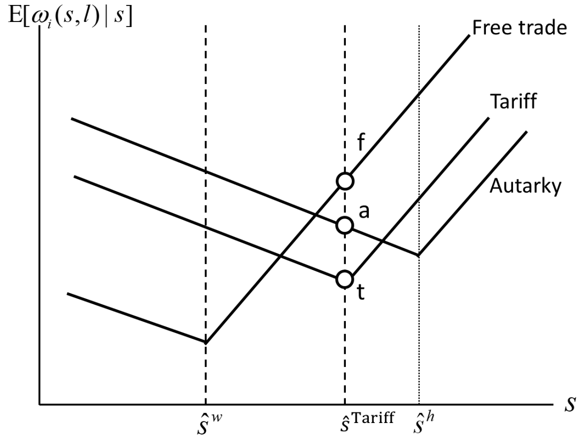
International trade also affects the wage income of workers who change their occupation. In Figure 4 (a), shifting from autarky to free trade, the income of workers with $\hat{s}^w < s < \hat{s}^h$ becomes higher in Industry 2 than in Industry 1. Accordingly, a worker will switch from the latter to the former. Interestingly, international trade has asymmetric effects on workers with $\hat{s}^w < s < \hat{s}^h$. In Figure 4 (b), point e is the intersection of wage profiles in autarky (solid line) and open economy (dashed line), and point e' is the skill combination that parallels e. The income of workers with $\hat{s}^w < s < e'$ is lower under free trade than in autarky. On the other hand, the income of workers with $e' < s < \hat{s}^h$ is higher than in autarky. For workers with $\hat{s}^w < s < e'$, it is better to move from Industry 1 to Industry 2 under free trade. If these workers were to remain in Industry 1 under free trade, their wage income would decrease. The reason for asymmetric effects on occupation change is related to the mismatching of the skill combinations between firms and workers. Since Home has a comparative advantage in Industry 2, the workers with $\hat{s}^w < s < e'$ change occupations to Industry 2. However, Industry 2 is not better suited to the workers with $\hat{s}^w < s < e'$. The mismatch between firms in Industry 2 and the workers increases. Therefore, the income of the group with $\hat{s}^w < s < e'$ decreases. In other word, there are winners and losers from international trade within industry.

Similarly, Foreign has the winners and the losers among those who change occupation in an open economy. Since Foreign has a comparative advantage in Industry 1, the workers with $\hat{s}^f < s < \hat{s}^w$ shift occupations to Industry 1. As seen in Figure 4 (b), under free trade, the wage income of workers with $\hat{s}^f < s < e'$decreases and that of $e' < s < \hat{s}^w$ increases, as compared with autarky. This result is similar to the case in Home. Hence, I find that international trade increases the skill combination mismatch between firms and workers, which results in a decrease in the wage income for a certain portion of the shifting workers; this result is not found in O&T.

Gradual Trade Liberalization

So far, we have considered trade liberalization as a discrete shift from autarky to free trade. However, each country does not shift from autarky to free trade. They impose some tariffs or non-tariff barrier on various goods. Recently, most countries reduced these trade barriers and the several prices approach to the price under free trade. Here, I assume a small country for simple. Suppose that the country imports good 1 and impose tariff on imported good 1. The gradual trade liberalization changes the relative price of goods, which affects workers' wage income. Figure 5 depicts the comparison of wage income among the three cases: autarky, tariff and free trade. $\hat{s}^{\text{Tariff}}$ is the threshold of occupation choice under tariff. Under autarky, workers with the skill combination $\hat{s}^{\text{Tariff}}$ are employed in Industry 1 and obtain wage income at Point a. When the country shifts from autarky to limited trade by tariff, the relative price changes and the threshold of occupation choice shifts from $\hat{s}^h$ to $\hat{s}^{\text{Tariff}}$. The workers with $\hat{s}^{\text{Tariff}}$ are indifferent with both industries since the wage income is the same in both industries. However, the worker obtains the new wage income in Point t, and this is less than that of autarky. The main reasons for the reduction in wage are the Stolper-Samuelson effects in Industry 1 and the skill mismatch in Industry 2. Suppose the country abolishes tariff barriers and begins free trade. In this case the relative price decreases and the threshold of occupation choice shifts from $\hat{s}^{\text{Tariff}}$ to $\hat{s}^W$. After free trade, the workers with $\hat{s}^{\text{Tariff}}$ completely move to Industry 2 because of the higher wage. These workers obtain the new wage income at point f, which is higher than that of limited tariff. When the shift to free trade sufficiently decreases the relative price, the negative effects by skill mismatch in Industry 2 are partly offset. Consequently, the wage income in parts of workers employed in Industry 2 may increase. Furthermore, the wage income under free trade is larger than that of autarky (a<f). If the government in the country decides to shift from autarky to complete free trade, the workers with $\hat{s}^{\text{Tariff}}$ may agree with the trade policy. However, the limited trade policy (i.e., tariff) is the more possible policy chosen by the government than free trade. If the government decides to shift from autarky to limited trade, the workers with $\hat{s}^{\text{Tariff}}$ may go against because of decreasing wage income (a>t). Workers whose skill sets are close to the threshold, alter their stance based on the kind of trade policy.

Figure 5. Gradual Trade Liberalization and Wage Inequality



3.2 *Role of* μ

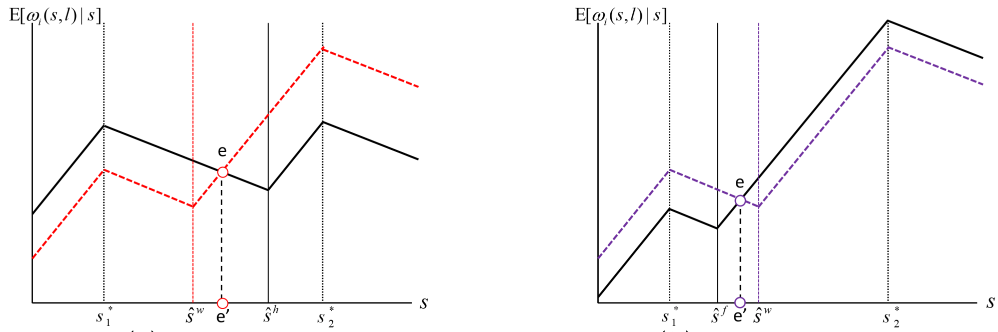
This section analyzes the role of other moments of the normal distribution, μ and $\sigma_j, j = s, l,$ on industrial structure. Since μ is the mean of s, the country with higher μ has more workers with a higher proportion of s-related skills. Therefore, a county with high μ produce

s-intensive goods more than the other country.[1] The cross-country difference in μ is also a source of international trade between countries.

Next, I examine the effects of μ on wage inequality. Suppose that there are two countries: Home and Foreign. From Eq. (5), the change in μ only affects the quantity of the expected wage income. In a country with high μ, wage income is higher than in other countries. Since μ also affects the relative price p through international trade, worker occupations and wage inequality are affected by it.

Figures 6 (a) and (b) describe the relationship between the expected wage profile and μ. Figure 6 (a) indicates the wage profile of the home country. Since the home country has higher μ, its autarkic relative price p^h is larger than that of the foreign country p^f. When international trade commences, the world relative price p^W is determined between p^f and p^h. In the home country, international trade with the foreign country decreases the relative price. The number of workers in Industry 2 increases, while that in Industry 1 decreases. Moreover, the reduction in the relative price raises the wage income in Industry 2, but decreases that in Industry 1. On the other hand, the foreign experiences an increase in the relative price by free trade. It increases the number of workers and their relative wage income in Industry 1 as compared to Industry 2.

Figure 6. Wage and Profile in the Home Country



(a) Home Country **(b) Foreign Country**

Similar to the analysis of ρ, free trade asymmetrically affects wage income for workers who change occupations. The workers with $\hat{s}^W < s < \hat{s}^h$ in Figure 6 (a) are those who change jobs after free trade. Among them, some experience an increase in the wage income while others receive a lower wage income. The workers whose income decreases are those who have the comparative advantage in the communicational skill l. Although shifting into Industry 2 is the best choice for them under free trade, it increases the skill mismatch between firms in Industry 2 and the workers. In the foreign country, the same situation occurs. As seen in Figure 6 (b), international trade makes workers with $\hat{s}^f < s < \hat{s}^W$ move to Industry 1.

[1] See Appendix 3.

However, some of the moving workers have a comparative advantage in quantitative ability h. The shifting of occupation may increase the divergence between the firm's demand and worker's skill set. Again, I find that a portion of the shifting workers will see a reduction in earnings as compared to autarky.

3.2 Role of $\boldsymbol{\sigma_s}$ and $\boldsymbol{\sigma_l}$

Next, we analyze the effect of the second moment, σ_s and σ_l, on international trade. There are two cases to observe the role of $\sigma_j, j = s, l$. To keep as equation and Figure 2-8 the same, I cannot change some conditions: any means $(\mathrm{E}(s) = \mu, \mathrm{E}(l) = 0)$, and any conditional means ($E(l|s) = \rho\sigma_l l(s - \mu)/\sigma_s$) and $E(s|l) = \mu + (\rho\sigma_s l/\sigma_l)$). Appendix 4 describes the relationship between variances and production possibility frontier. From the appendix, I show the change in σ_s and σ_l do not affect production structure, and occupation choice for workers. Therefore, there is no trade between countries which only have different variances of skills.

4 Conclusions

In this study, I analyzed the role of two-dimensional skills and skill matching on international trade and wage inequality. I assumed that individuals have two primary skills and that each individual has a different combination of the skill set. I also assumed that the patterns of these skill combinations were normally distributed and each industry had a demand for a certain combination. However, it was difficult for workers to find an ideal industry which matches with their skill combination. Therefore, the workers are employed in the industry which is unsuited to their skill combinations. I referred to this as a mismatch in the labor market. The distribution of skills also affected the industry structure and within-industry income inequality. I also considered two countries with different skill distributions and examined the effects of free trade. The differences in the cross-country distributions of skills led to international trade. When international trade commenced, a portion of the workers employed in an importing industry moved to an exporting industry, as they could earn higher wages than if they remained in the former. However, I found that some of these workers who shifted occupations were matched with less appropriate firms and received lower wages than in an autarky.

Very few theoretical and empirical studies have focused on multi-dimensional skills and international trade. My analysis provides additional insights into the existing literature in this field. I also elucidated the role of the labor market on international trade and provided implications for more effective labor policies. An interesting extension of this paper would introduce unemployment and other labor market imperfections to this type of analysis.

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Appendix 1: $\boldsymbol{Y_1}$ and $\boldsymbol{Y_2}$

To describe PPF, this section shows the detailed calculation of output of industry 1 and 2. I describe the industry outputs of two sectors considering occupation choice. Industry output is the sum of two industries that each labor creates with their skill combination, y_i.

$$Y_1 = \int_{-\infty}^{\hat{s}} \int_{-\infty}^{\infty} e^{\ln y_1} f_{sl}(s,l) \mathrm{d}l \mathrm{d}s$$

$$= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} \mathrm{e}^{\ln s + l - \ln \beta_1} f_{sl}(s,l) \mathrm{d}l \mathrm{d}s + \int_{s_1^*}^{\hat{s}} \int_{-\infty}^{\infty} \mathrm{e}^{l - \ln(1-\beta_1)} f_{sl}(s,l) \mathrm{d}l \mathrm{d}s \qquad (A1)$$

$$Y_2 \equiv \int_{\hat{s}}^{\infty} \int_{-\infty}^{\infty} \mathrm{e}^{\ln y_2} f_{sl}(s,l) \mathrm{d}l \mathrm{d}s$$

$$= \int_{\hat{s}}^{s_2^*} \int_{-\infty}^{\infty} \mathrm{e}^{s + l - \ln \beta_2} f_{sl}(s,l) \mathrm{d}l \mathrm{d}s + \int_{s_2^*}^{\infty} \int_{-\infty}^{\infty} \mathrm{e}^{l - \ln(1-\beta_2)} f_{sl}(s,l)\, \mathrm{d}l \mathrm{d}s \#(\mathrm{A2})$$

where $f_{sl}(s,l)$ is the probability density function. For the detail, I integrate by respect to l with using the marginal distribution of s, f_s. Y_1 and Y_2 have two terms of integration. Since each integration is complex, we calculate each term individually until they need a different way to calculate.

The first term of Eq. (A1) becomes:

$$= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} \mathrm{e}^{s + l - \ln \beta_1} \frac{f_{sl}}{f_l} f_s \mathrm{d}l \mathrm{d}s$$

$$= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} \mathrm{e}^{l} \frac{f_{sl}}{f_s} \mathrm{d}l \cdot \mathrm{e}^{s - \ln \beta_1} \mathrm{d}s$$

$$= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} \mathrm{e}^{l} \frac{1}{\sqrt{2\pi}\sigma_l \sqrt{1-\rho^2}} \exp\left(-\frac{\left[l - \rho \left(\frac{\sigma_l}{\sigma_s} \right) (s - \mu) \right]^2}{2\sigma_l^2 (1-\rho)} \right) \mathrm{d}l \cdot f_s \mathrm{e}^{s - \ln \beta_1} \mathrm{d}s$$

$$= \int_{-\infty}^{s_1^*} \frac{1}{\sqrt{2\pi}\sigma_l\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left(- \frac{\left[l - \frac{\sigma_l}{\sigma_s}\{\rho(s-\mu) + \sigma_s\sigma_l(1-\rho^2)\}\right]^2}{2\sigma_l^2(1-\rho^2)} \right) \mathrm{d}l$$

$$\cdot \exp\left[\frac{\sigma_l}{\sigma_s}\rho(s-\mu) + \frac{\sigma_l^2}{2}(1-\rho^2) \right] f_s \mathrm{e}^{s - \ln\beta_1}\,\mathrm{d}s$$

Since $\frac{1}{\sqrt{2\pi}\sigma_l\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left(- \frac{\left[l - \frac{\sigma_l}{\sigma_s}\{\rho(s-\mu) + \sigma_s\sigma_l(1-\rho^2)\}\right]^2}{2\sigma_l^2(1-\rho^2)} \right) \mathrm{d}l$
is a normal probability distribution which has a mean of $\sigma_s\sigma_l(1-\rho^2)$ and variance of $\sigma_l(1-\rho^2)$, I set this part as 1. Hence,

First term of Y_1

$$= \int_{-\infty}^{s_1^*} \exp\left(\frac{\sigma_l}{\sigma_s}\rho(s-\mu) + s - ln\,\beta_1 + \frac{\sigma_l^2}{2}(1-\rho^2) \right) \cdot f_s\,\mathrm{d}s$$

$$= \frac{1}{\sqrt{2\pi}\sigma_s} \int_{-\infty}^{s_1^*} \exp\left(s - ln\,\beta_1 + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2 \right) \mathrm{d}s$$

Similarly,

Second term of Y_1

$$= \frac{1}{\sqrt{2\pi}\sigma_s} \int_{s_1^*}^{\hat{s}} \exp\left(-\ln(1-\beta_1) + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2 \right) \mathrm{d}s$$

First term of Y_2

$$= \frac{1}{\sqrt{2\pi}\sigma_s} \int_{\hat{s}}^{s_2^*} \exp\left(s - \ln\beta_2 + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2 \right) \mathrm{d}s$$

Second term of Y_2

$$= \frac{1}{\sqrt{2\pi}\sigma_s}\int_{s_2^*}^{\infty} \exp\left(-\ln(1-\beta_2) + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2\right) \mathrm{ds}$$

Appendix 2: Industry structure and ρ

To investigate the relationship between ρ and the industry output, I conduct numerical simulations and observe the change of Product Possibility Frontier (PPF) for two industries. The PPF describes the industry structure in an economy and provides us an intuitive interpretation about international trade. To engage in numerical simulation, we set the specific values of some parameters. I use the following parameters as long as each parameter does not play a role in the analyses.

$$\beta_1 = 0.2$$

$$\beta_2 = 0.8$$

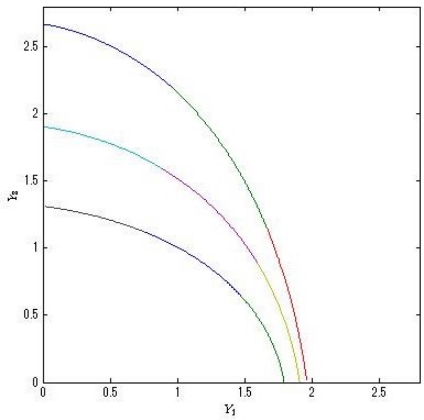
$$\mu = 0 \qquad \# \text{(A3)}$$

$$\rho = -0.5$$

$$\sigma_s = \sigma_l = 1$$

Figure 7 plots the comparison of PPF with respect to ρ. The higher the ρ is, the more the PPF curves shift outside, and note that the expansion of PPF is biased to industry 2. The effect of ρ is similar to the Rybczynski theorem.

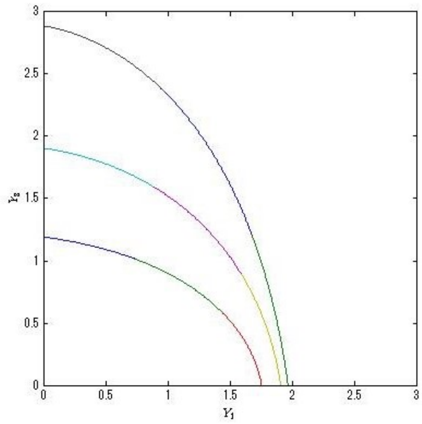
Figure 7. Production Possibility Frontier and ρ



Appendix 3: Industry structure and μ

Figure 8 plots the comparison of PPF with respect to μ with the parameters in Eq. (A3). The higher the μ is, the more the PPF curves shift outside, and note that the expansion of PPF is biased to industry 2. The effect of μ is also similar to the Rybczynski theorem.

Figure 8. Production Possibility Frontier and μ



Appendix 4: Industry structure and σ

I use the following three cases of σ_s: (i) $\sigma_s = \sigma_l = 1$ (benchmark), (ii) $\sigma_s = \sigma_l = 0.8$, and (iii) $\sigma_s = \sigma_l = 1.2$. The figure 9 depicts the relationship between variances and PPF. The PPF located at the lowest position corresponds to (ii), that at the highest position corresponds to (ii), and that located in the middle corresponds to (ii). Larger variance depicts larger production possibility frontier. As you see, the output of both industries expands equivalently as σ_s and σ_l increases.

Figure 9. Production Possibility Frontier and σ_s and σ_l