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Demand Elasticity in the IPEX: Bayesian Experiment under Dual Pricing Scheme

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Abstract: This study runs an experiment aiming to provide a flexible and analytically elegant framework to reliably estimate the price elasticity of electricity demand. Inference pertains to the demand at hourly level in the Italian wholesale electricity market and uses individual demand bid data. Individual bids represent the ex-ante willingness to pay and thus allows for constructing a market demand grounded in the consumer behaviour theory, by exploiting the duality approach.

The Bayesian econometric estimation is applied, relaxing homoskedasticity assumptions of the traditional linear regression model. It allows to identify robust results, showing that elasticity varies significantly among hours of the day, zone segmentations as well as the level of equilibrium prices. Bayesian inference provides also the opportunity to include prior information sourced from previous studies and the institutional structure governing the agent behaviours. This prior information involves some degree of uncertainty, for this reason the Bayesian approach defines for it a probability distribution. Using Bayes rule, prior information is then updated according to the observed data.

Results show that market reforms and time-varying pricing schemes trigger the consumers' price reaction leading to a price elasticity different from zero.

JEL classification: D43; L13; L81; Q49.
Keywords: Demand Elasticity; Bayesian Heteroskedastic SUR Model; Power Market.

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1 Introduction

Since the early 1970s, when energy caught the attention of policymakers in the aftermath of the first oil crisis, research on electricity demand has vastly increased in order to overcome the limited understanding of the nature of the energy demand and demand response due to the presence of external supply shocks. Most of the early papers dealt with flat electricity rates in the context of vertically integrated monopolies. The worldwide deregulation of power industry has created new challenges to the demand side management. After deregulation, electric utilities restructured their operations from vertically integrated mechanisms to open market systems. Moreover, the strong and constant increase in energy consumption has imposed an accurate planning in order to avoid electricity shortage and guarantee adequate infrastructures; many consumer rate programs have been implemented to promote a greater demand response to price, and thus a more efficient electricity market. Consequently, price elasticity estimation has received more attention in the recent literature as it become an essential feature for energy planning, formulating strategies and recommending energy policies.

Reforms have involved also the day ahead Italian electricity market (DAM), where hourly blocks of electricity are exchanged in advanced (until the day before) respect to the time of the injection into the grid. DAM is organized according to an implicit double auction mechanism where bids/offers are accepted under the economic merit-order criterion; these bids represent the ex-ante willingness to sell/pay electricity and compete to form the hourly market supply/demand. In this context, this study aims to analyse the buyers' response in the DAM under dual competitive pricing scheme for 2011. The analysis focuses on 2011 for two reasons. First, the 2011 immediately follows the implementation of a dual (peak-off peak) retail price structure for residential consumers, introduced by the Regulatory Authority in 2010. This new double structure is particularly strategic for this study that aims to analyse how the consumers' prices response has changed since the introduction of the new time-varying pricing scheme. Second, 2011 was the first year in which renewable (mainly photovoltaic) capacity increased dramatically from below 9 GW to 19.7 GW, inducing considerable peak-shaving and modifying peak-off peak price differentials.

Since the widespread wave of power market reforms, literature has been enriched by several papers on the estimation of price elasticity demand for electricity, focusing both on short- and long-term, applying different techniques, models and data, and thus, discussing most of the critical aspects of the new competitive markets. However, some issues have remained to be explored. First, most of the empirical analyses on electricity demand have used aggregated data of market quantities and prices or suppliers' bids. In all these cases, the "demand" behaviour is estimated from "sup-

ply" data, while there is a more direct and obvious way to estimate the demand behaviour, i.e. using individuals' bid data. Second, most studies have assumed that hourly demands work independently. However, the possible correlation among individuals' electricity demands referring to different hours cannot be ignored. Inside the different time slots, consumers (both industrial and domestic users) can in fact handle and regulate their electricity demands. Each time slot characterizes different economic activities, habits and consumption needs, which buyers satisfy through load profiles which are spread over the several hours of the slot. Last, no studies have applied the Bayesian framework, that allows to enrich the statistical model with more features. Bayesian inference provides in fact the opportunity to include the information and the general knowledge collected during the previous institutional experiences or experiments. This information is called prior information. Prior information involves some degree of uncertainty as well, for this reason the Bayesian approach assigns to it a probability distribution.[1] Since there is no certainty on the value of model's parameters to estimate, the Bayesian inference assigns to them a prior probability distribution. This prior distribution is then combined with the distribution of data (the likelihood function) exploiting the Bayes rule. The result of the inference is the posterior distribution of parameters, that represents the updated knowledge about the parameter values.
Three are the distinct features of this paper. Firstly, it uses micro level data, exploiting information provided by buyers' ex-ante bids, that represent their real willingness to pay. This allows for constructing an empirical demand for the Italian power market based on the optimizing microeconomic model. Secondly, it defines more flexible assumptions about consumer behaviour, whereby buyers schedule their daily loads among the two different group of hours (peak and off-peak), bearing in mind possible changes in the market prices and thus changing their bids. The assumption of independent hourly bids is thus relaxed, and the hourly demand equations are correlated across the two groups. SUR model with heteroskedastic error terms well fits with this assumption. Thirdly, study uses the Bayesian method, more robust and flexible than the frequentist one, since it treats heteroskedastic SUR model as a hierarchical one. In the Bayesian framework, the assumption of heteroskedastic and correlated error terms is in fact equal to assuming them distributed according to the more general mixture of Normals.
Paper is structured as follows. Section 2 quickly summarizes the literature's state of art while Section 3 describes data and the theoretical framework. Section 4 offers a brief digression about the Bayesian methodology applied,

[1] It is noteworthy that the probability in the Bayesian approach is based on the subjective definition (De Finetti, 1993), according to which probability is a measure of individual degree of belief relying on relevant experiences. The subjective definition tries therefore to build up a formal probability theory on behaviouristic basis.

while empirical results are shown in Section 5. Section 6 concludes and summarizes the main findings.

2 State of the Art

Large number of recent academic studies estimate the price elasticity demand for electricity, and most of these studies clearly demonstrate the significance of the price elasticity of both industrial and residential demands. One pioneer study on the elasticity for residential consumer is Caves and Christensen (1980), who estimated the Wisconsin demand elasticity under time-of-use electricity pricing experiment. Authors found a clearly discernible peak/off-peak shifts of kilowatt-hour consumption in response to the time of use pricing.
Filippini (1995) estimated the price and expenditure elasticity of peak and off-peak electricity demand applying the Almost Ideal Demand System model (AIDS) to a micro-dataset on Swiss households. He found that the peak consumption was more responsive to the peak pricing than the overall consumption was to an averaged price index. This was due to the incentive to substitute between peak and off-peak consumption under differentiated tariffs.
Patrick and Wolak (2001) estimated the electricity demand purchased by industrial and commercial customers according to half-hourly energy prices in the England and Wales electricity markets. They found that price elasticities varied considerably across industries, as did the pattern of within-day substitution in electricity consumption.
King and Chatterjee (2003), Reiss and White (2005), and Faruqui and Stephen (2005) analysed the Californian wholesale electricity market. All these studies showed that the demand price response was significant and able to reduce peak-period loads under time-varying prices.
Wolak (2003) as well estimated demand elasticity of the Californian electricity market to measure the unilateral market power of the five largest electricity suppliers. Using the suppliers' bids he computed the hourly price elasticity of the ex post residual demand curve faced by each supplier and found a significant market power exercised by firms. In this previous study, the demand elasticity was estimated from "supply" data, while in this paper a more direct and obvious approach is used to estimate demand behaviour, that is the employment of information from individuals' bid data.
Taylor et al. (2005) investigated the real time pricing programs in the UK wholesale electricity market and found that industrial customers gained experience with hourly pricing and reduced their loads during higher priced hours.
Nahata et al (2007) estimated the demand elasticities for households and industrial users in the Russian electricity market in order to find the best

social pricing scheme.[2]
In a more recent paper, Wolak (2010) estimated the residual demand for each market participant in an Cournot electricity market. Using dynamic pricing experiment that compares the performance of different programs in the District of Columbia, he showed that customers were price responsive and substantially reduced their electricity consumption during high-priced periods.
Lin and Lui (2011) tested the effects of the differential power pricing policy in the Chinese electricity market. Using Ramsey model, they showed that industrial consumers decreased total costs and improved their productive efficiency in advance, as they anticipated higher electricity tariffs.
Kamyab and Bahrami (2016) proposed a scheduling algorithm to manage electricity consumption of energy hubs on the customer side. The interaction among energy hubs was modelled under time of use and dynamic pricing schemes functioning in a competitive market. They showed that algorithm are effective for both costumers and companies, reducing the daily cost for about 10%.
Meta-analyses on price elasticity of electricity demand was conducted by Espey and Espey (2004) and Labandiera et al. (2017). Both studies tried to identify the factors that systematically affect elasticity finding significant differences in the price responsiveness between short and long-run.

3 Material and Methods

The following Bayesian experiment treats individuals' bid data from DAM. DAM is an wholesale market where hourly blocks of electricity are negotiated and hourly prices and volumes are defined through the intersection between demand and supply curves. In this market, electricity is traded by scheduling generating and consuming units.
Market participants submit bids/offers where they specify the volume and the maximum/minimum price at which they are willing to purchase/sell electricity. DAM is organized according to an implicit double auction mechanism where bids/offers are accepted under the economic merit-order criterion and subjected to transmission limits between zones. Supply offers are ranked in an increasing price order on an aggregate supply curve, while demand bids are ranked in a decreasing price order on an aggregate demand curve. The intersection of the two curves gives: the overall traded volume, the clearing price, the accepted bids/offers, and the injection and withdrawal schedules obtained as the sum of the accepted bids/offers. The market algorithm will accept bids/offers in such a way as to maximize the value of transactions, given the maximum transmission limits between

[2] Authors used Ramsey model to show that prices are not socially optimal. A decrease in price for industrial users and an increase in price for households would bring prices closer to the socially optimal.

zones.
The equilibrium price in DAM is set by the system marginal price (SMP). Accepted supply offers are remunerated at the Zonal Clearing Price, while accepted demand bids are remunerated at the National Single Price (PUN), that is the weighted average of zonal supply prices. The accepted offers/bids determine the preliminary injection and withdrawal schedules of each offer point for the next day. The demand side is essentially represented by industrial demand (natural or legal persons entitled to choose their own supplier of electricity producer, distributor and wholesaler) and traders, while the Single Buyer covers the demand of captive customers. Industrial consumers use power as an input in the production function to produce goods and services, while residential consumers have a domestic use of electricity. Two critical aspects need to be investigated: the heterogeneity of consumers, and the presence of both Single Buyers and traders that may exercise opportunistic behaviours.
Firstly, the presence of heterogeneous consumers reflects different price responses. Some consumers express a quantity bid without specifying the price they would be willing to pay. These consumers show an ex-ante perfect inelastic behaviour, as they are (in principle) willing to pay any price that will result from the market clearing procedure.[3] Other consumers specify instead both quantity and price bid and, in turn, they have to be considered elastic consumers.
Given consumers' heterogeneity, bids are divided in two groups representing the two different price responses.

- Bids with no price are gathered into a group with non-elastic demand $y_1 = f(p_1)$ with elasticity $\epsilon_1 = 0$.

- Bids with price refer instead to an elastic consumer (aware of the wholesale market price signals) with demand $y_2 = f(p_2)$ and $\epsilon_2 < 0$.

The two kinds of consumers also differ on their reserve prices $p_2^\star = f_2^{-1}(0) < p_1^\star = f_1^{-1}(0)$. Given the equilibrium price $p^\star$, the aggregate demand can fall into two cases:

- If $p_2^\star < p^\star \leq p_1^\star$, then the market demand is $y = y_1 = f_1(p_1^\star)$, the market demand is expressed only by group 1 and $y_2 = 0$.

- If $p_1^\star > p_2^\star \geq p^\star$, then the market demand is $y = y_1 + y_2 = f(p^\star)$, the market demand is given by the aggregation of the demands of both types of consumers.

[3] The DAM assigns a default price limit to these bids, set equal to the maximum price cap imposed to suppliers by the Regulatory Authority. The default price assigned to these bids has increased over time from a level of 200 euro/MWh in 2004 to 3000 euro/MWh in 2009-2011

When all individual bids are aggregated in a market demand function, the curve is vertical until the portion of "elastic consumers" is reached.
Secondly, agents who submit demand bids are not necessarily the final users of electricity. Single Buyers and traders are intermediary agents which demand electricity on the behalf of final customers. How the role of Single Buyer and traders should be processed into the model is an open question. The contractual nature of trader-customer relationship suggests that this can be treated within the perspective of the principal-agent relationship, where consumer is the principal and trader is the agent.[4] Specifically, the principal may have some private information, e.g., in deciding to save energy during the day, to use air conditioning in response to local weather shocks, as well as in altering daily consumption profile due to labour strikes or to meet sudden demand shocks. Agents can develop statistical forecasting models of their customer behaviours based on their customers' characteristics. In this case, the mild requirement that agents follow individual rationality and incentive compatibility allows to find a Pareto optimal equilibrium where all types of traders and customers are engaged in a contract which maximizes their respective utility functions.
Other opportunistic behaviours, such as arbitraging between DAM and Infra-Day Market, may rise from the SMP that imposes purchasers to pay a PUN price different from the price they bid. This may give rise that agents exercise some market power on the demand side.[5] However, this latter possibility is removed away for two reasons. The first one is structural: the number of traders and consumers in the market is quite large and their market share is low, therefore, market on the demand side is substantially competitive. The second one is institutional: Regulatory Authority imposes penalty charges if the real loads deviate from the withdrawal profiles defined in the DAM. Therefore, traders have to submit bid reflecting the real willingness to pay. Since agents incentives are consistent with principals interests, the derived equilibrium is Pareto-Optimal.
Industrial consumers choose the amount of electricity input which minimizes their cost function given the technological constraint, while residential customers are part of optimizing utility function process given the budget constraint. Therefore, this econometric approach lies inside the neoclassical framework and is grounded on rational optimizing behaviour theory. Since data refers to hourly bids, the duality approach gives the theoretical justification to legitimately switch from agent's preferences (optimization theory) to the Marshallian demand, where quantities are functions of prices and total expenditure. In each hour of the day, all the agents taking part in the DAM rationally behave minimizing a cost function C, production cost function for industrial buyers and expenditure function for the residential ones.

[4] See Maskin and Tirole (1992) and Bigerna and Bollino (2014)
[5] See Wolak (2001).

FOCs derive the Hicksian demands:

$$\frac{\partial C(Q, p_i, p_{-i})}{\partial p_i} = h_i(Q, p_i, p_{-i}) = q_i \text{ for all } i \tag{1}$$

where Q represents the firms output or consumers utility and p is the price vector, p_i is the price of electricity for the hour i, p_{-i} is the price vector of all other goods.
Exploiting the homogeneity and separability properties of cost function and applying the Roy identity, it is possible to switch from the Hicksian demands h_i to the Marshallian demands y_i:

$$\frac{\partial C(Q, p_i, p_{-i})}{\partial p_i} = h_i(Q, p_i, p_{-i}) = y_i(m, p_i, p_{-i}) \tag{2}$$

$y_i(m, p)$ expresses hourly market demand for electricity as a function of its own price p_i, the prices of all the other goods p_{-i} and the budget constraint. The multidimensional model needs to be reduced into a bi-dimensional problem. For this reason, all the other goods and inputs will be bundled in a numeraire good. The numeraire is evaluated at a price proxied by the monthly consumer price index $\bar{p}$ (adjusted excluding from its computation the energy consumption).
It is assumed that electricity demand profiles substantially differ within the day. Hours between 9 a.m. and 8 p.m. (though as the peak hour group) are assumed being characterized by the prevalence of business activities, while hours between 9 p.m. and 8 a.m. (defined as the off-peak hour group) being characterized by the prevalence of domestic uses of electricity. Table 1 reports the monthly summary statistics for the equilibrium market prices and quantities and confirms this assumption. The total quantity submitted in a off-peak hour was on average 25% lower than the total quantity recorded in a peak hour. Differences can be noticed also in the PUN: during peak hours, as demand was higher, equilibrium price was higher.[6]
Therefore, day is divided into two groups (peak and off-peak hours), within these two groups of hours, individuals' demands are correlated because participants can change their loads and affect their price responses. The Seemingly Unrelated Regression (SUR) model well fits with this assumption. SUR model is a multiple equations regression model, in our case, the equations are 12, one for each hour within the group. The hourly electricity demand in a SUR model is the following:

$$y_{m,i} = \alpha_m + \beta_{m,p}\left(\frac{p_{m,i}}{\bar{p}}\right) + \sum_{k=3}^{K} \beta_{k,m} d_{k,m,i} + \epsilon_{m,i} \tag{3}$$

where $m = 1, ..., M$ the hour equations and $i = 1, ..., N$ denotes observations. $M = 12$ represents the total number of hours within the same group.

[6] These statistics are consistent with the previous empirical evidences where consumers are responsive to time-varying prices.

Table 1. Summary Statistics for the Market Variables.

	January	February	March	April	May	June	July	August	September	October	November	December	Average
							PUN Peak						
Mean	73.09	75.50	77.29	70.91	76.50	73.65	76.02	77.52	88.63	85.73	90.12	91.77	79.73
Min	54.00	55.79	53.65	47.83	47.45	44.40	31.00	47.00	57.59	50.22	45.90	59.21	49.50
Max	91.72	110.40	142.96	118.07	98.01	102.83	142.67	123.58	123.54	151.09	160.62	164.80	127.52
							PUN Offpeak						
Mean	56.92	57.07	59.06	59.45	66.05	63.18	63.45	71.50	73.99	71.49	66.82	66.96	65.14
Min	10.00	27.00	36.18	22.94	39.98	37.49	16.14	31.06	40.01	23.53	28.00	28.00	28.36
Max	84.07	90.37	115.13	105.33	97.43	100.24	101.78	132.99	131.71	137.84	115.66	133.16	112.14
							Quantity						
mean Peak	41865	43244	40983	36767	37571	39353	42169	35166	40148	39132	41002	40035	39786
Mean Off-Peak	31379	32682 .33	31807	30135	30314	31615	34330	29645	32502	31246	31084	30348	31424
Diff	10487	10562	9177	6632	7257	7738	7838	5522	7646	7886	9918	9687	8363

The model uses a log-linear demand function: the dependent variable $y_{m,i}$ is the logarithm of the hourly electricity demand, the explanatory variables are the corresponding logarithm of price, $p_{m,i}$, adjusted by the monthly consumer index price $\bar{p}$ and variables $d_{k,m}$ that refer to a group of socio-economic determinants proxying the real total expenditure and the scale effect (i.e. the daily and zonal intercept dummies).
Let stack observations:

$$\underbrace{y_m}_{N\times 1} = \underbrace{X'_m}_{N\times K}\underbrace{\beta_m}_{K\times 1} + \underbrace{\epsilon_m}_{N\times 1} \tag{4}$$

$$\epsilon \sim N(0, \Sigma_m) \tag{5}$$

where: $y_m = [y_{m,1}, ..., y_{m,N}]$;
X_m is the staked matrix of the vectors of explanatory variables $\Big[1, \left(\frac{p_{m,i}}{\bar{p}}\right), d_{3,m},$ $..., d_{K,m}\Big]$;
β'_m is the $(K \times 1)$ vector of parameters $\beta_m = [\alpha_m, \beta_{m,p} ..., \beta_{m,K}]'$ for $m = 1, ..., M$;
$\epsilon_m = [\epsilon_{m,1}, ..., \epsilon_{m,N}]$;
Σ_m is the $(N \times N)$ variance-covariance matrix.
We assume that collusive behaviours among buyers are not admitted as they buy electricity independently without rearranging their bids according to the bids of others. Therefore, Σ_m is a diagonal matrix.

Stacking further, the model can be written in a more compact form:

$$y = X\beta + \varepsilon \tag{6}$$
$$\epsilon \sim N(0, \Omega)$$

where:
$y = [y_1, y_2, ..., y_M]'$ for $i = 1, ..., N$ is the $(NM \times 1)$ stacked vector of y_i;
$X = [X_1, X_2, ..., X_M]'$ for $i = 1, ..., N$ is the $(NM \times K)$ stacked matrix of X_m;
$\epsilon = [\epsilon_1, \epsilon_2, ..., \epsilon_M]'$ for $i = 1, ..., N$ is the $(NM \times 1)$ stacked vector of ϵ_m;
Ω is any positive $NM \times NM$ matrix that can be expressed in terms of precision h^{-1} and matrix Λ^{-1}: $\Omega = h^{-1}\Lambda^{-1}$. The non-zero covariance matrix Ω implies that the M equations can be related and regressions are tied into a system of equations to analyse together. This assumption wants to model the hypothesis that market participants have the opportunity to reprogram their activities and reschedule their demand profiles (within the group of hours). Moreover, the M variances can also differ.
Model is hierarchical, since matrices Λ_m are unknown. Lastly, since Ω is a positive definite matrix Cholesky decomposition is applied, that is it exists a $(NM \times NM)$ matrix P such that $P\Omega P = I_{NM}$. Model can be transformed multiplying both sides of equation by P and obtaining:

$$y^\star = X^\star\beta + \epsilon^\star \tag{7}$$

where $y^\star = Py$, $X^\star = PX$ and $\epsilon^\star = P\epsilon \sim N(0, I_{NM})$

4 Bayesian Methodology

4.1 Prior Distributions

The Bayesian framework allows assessing which random mechanism generates data, given the observed sample. The parameters indexing the probability distribution generating data are considered as random variables. This randomness expresses the uncertainty about the true values of parameters and this uncertainty is modelled assigning to them a probability distribution. This distribution is called prior and it is subjective as it originates from pre-experimental information. In this model parameters to be estimated are β, h and Λ. The prior distributions of parameters are independent, that is:

$$p(\beta, h, \Lambda) = p(\beta)p(h)p(\Lambda) \tag{8}$$

For β and h the prior distributions are Normal and Gamma, respectively:[7]

$$\begin{aligned} p(\beta, h) &= p(\beta)p(h) \\ p(\beta) &= f_N(\beta_0, V_0) \qquad (9) \\ p(h) &= f_G(\nu_0, s_0^{-2}) \qquad (10) \end{aligned}$$

The prior hyperparameter elicitation for β comes from the previous empirical study of Bigerna and Bollino (2014), the β's Normal Prior distribution is centered on the frequentist hourly estimates referring to the previous year (2010).[8]

Introducing the unknown heteroskedasticity Λ can causes dimensionality problems, increasing dramatically the number of parameters to estimate. If Λ_m were treated as completely independent and unrestricted matrices, there would not be enough observations to estimate all parameters (equal to $N\left(\frac{M(M-1)}{2}\right) + k + 1$). Therefore, exchangebility property is assumed (Koop,

[7] Usually Bayesian technique suggests to use natural conjugate priors, where the β's distribution be dependent Ω, in this way the joint posterior distribution would become: $p(\beta, h) = p(\beta|h)p(h)$. This joint prior has the advantage to derive analytically tractable joint posterior distributions whose main summary statistics are available, ruling out the use of posterior simulator. However, the natural conjugate prior for the SUR model has been found by many to be too restrictive. The prior covariances between coefficients in each pair of equations are in fact all proportional to the same matrix. Therefore, in this study the independent Normal-Gamma prior is used.

[8] When there are no pre-experimental information, this model allows to use *non-informative prior*, simply setting $\nu_0 = 0$ and allowing to the prior variance of β (V_0) to go to infinity

2003):

$$p(\Lambda) = \prod_{m=1}^{M} p(\Lambda_m) \tag{11}$$

$$p(\Lambda_m) = f_W(I_N, \nu) \tag{12}$$

$$p(\nu) = f_G(\nu|25, 2) \tag{13}$$

This assumption puts some structure to data and allows for all Λ_m to be different from one another, but at the same time, to be drawn from the same Wishart distribution[9]. Thus, model is flexible, but enough structured for statistical inference.[10]
In this way normality assumption is relaxed, as unknown heteroskedasticity model is equivalent to a linear regression model with the Student-t errors Koop (2003).
Model involves in fact a mixture of Normal distributions that is a powerful tool in applied economics since does not suggest a particular form for the likelihood function.[11] Equation (15) assigns prior distribution to the degree of freedom ν as well. As suggested by Koop (2003), ν must be strictly positive, then it can follow a gamma distribution with prior mean equal to 25 and degree of freedom equals two.[12]

4.2 *Posterior Distributions*

After observing the data, the likelihood function is computed. The two main features of Bayesian inference, prior and likelihood, are then combined together using Bayes Theorem and lead to the so-called posterior distribution. The posterior is proportional to the prior times the sample likelihood. The Bayes Theorem describes how prior knowledge are updated by the sampling. It represents the distribution of the parameter after observing the experimental data. The joint posterior distribution describes our assessment of where the true values of parameters are likely to lie in the parameter space, after observing the sample. If the interest is on the point estimates,

[9] In this way, the number of parameter to estimate is $\left(\frac{M(M-1)}{2}\right) + k + 1$.

[10] Rather remarkably, this model turns into a linear regression model with i.i.d. multivariate Student-t errors with ν degrees of freedom; that is $f(\epsilon_m) = f_t(0, h^{-1}\Sigma_m, \nu)$ (Geweke, 1993). The Student-t distribution is the general case of the Normal distribution whit $\nu \rightarrow \inf$. Thus, model allows for a more flexible error distribution without leaving the Normal linear regression model framework.

[11] The assumption $H_0 = \nu \longrightarrow \infty$ against the alternative that ν is finite was tested using the Bayes factor approach proposed by Gelfand-Dey (Gelfand and Dey, 1994). Test rejected the null hypothesis confirming the t-Student model.

[12] Geweke (1993) noted how the choice of the parameter for ν's prior distribution is critical, since it can threaten the inference for β parameters. Setting the prior mean equal to 25, the prior weight is substantially assigned both to very fat-tailed error distributions (e.g. $\nu < 10$), as well as to roughly Normal distributions ($\nu > 40$).

the means of the posterior distributions are the natural choice. Hence, multiplying the conjugate prior with the likelihood function and discarding the constant terms leads to the following posterior distribution:

$$\begin{aligned} p(\beta, h, \Lambda, \nu|y) \propto\ & p(\Lambda|\nu)p(\nu) \\ & \times \exp\{\frac{-h}{2}[(y^{\star} - X^{\star}\beta)'(y^{\star} - X^{\star}\beta) \\ & + (\beta - \beta_0)'V_0^{-1}(\beta - \beta_0)]\} \\ & \times h^{\frac{N+\nu_0-2}{2}} \exp\left[-\frac{h\nu_0}{2s_0^{-2}}\right] \end{aligned} \tag{14}$$

This joint posterior density does not take any well-known form and and, hence, needs to be simulated. Gibb Sampling algorithm has to be applied and required the derivation of full conditional posterior distributions for all parameters β, h, Λ, ν.

$$p(\beta|y, h, \Lambda, \nu) = N(\beta_n, V_n) \tag{15}$$

$$V_n = \left(V_0^{-1} + hX'\Lambda X\right) \tag{16}$$

$$\beta_n = V_n\left(V_0\beta_0 + h^{-1}X'\Lambda X\hat{\beta}(\Omega)\right) \tag{17}$$

$$p(h|y, h, \Lambda, \nu) = G(s_n^2, \nu_n) \tag{18}$$

$$\nu_n = N + \nu_0 \tag{19}$$

$$s_n^2 = \frac{(y - X\beta)\Lambda(y - X\beta) + \nu_0 s_0^2}{\nu_n} \tag{20}$$

$$p(\Lambda|y, \beta, h, \nu) = \prod_{m=1}^{M} p(\Lambda_m|y, \beta, h, \nu) \tag{21}$$

$$p(\Lambda_m|y, \beta, h, \nu) = f_W((\nu + \frac{N+1}{2})\left[h\left[\epsilon_m\epsilon_m'\right] + \nu I_N\right]^{-1}, \nu + \frac{N+1}{2}) \tag{22}$$

ν does not enter the likelihood function, then $p(\nu|y, \beta, h, \Lambda) = p(\nu|\Lambda)$.[13] However, $p(\nu|\Lambda)$ does not take a well known form and requires to implement further Metropolis-Hasting algorithm within the Gibb sampling. The Metropolis-within-Gibbs algorithm is the following. Draws of β and h are taken from (17) and (20), respectively. Draws from $p(\Lambda_m|y, \beta, h)$ are taken using (24). For $p(\nu|\Lambda)$ a Random Walk Chain Metropolis-Hastings algorithm is used and Normal increment random variable is applied.[14]

[13] Note that, conditioning on Λ, ν adds no new information and, thus, $p(\beta|y, h, \Lambda, \nu) = p(\beta|y, h, \Lambda)$ and $p(h|y, \beta, \Lambda, \nu) = p(h|y, \beta, h, \Lambda)$.

[14] For more details on Bayesian Computation readers are invited to see Koop (2003), pp. 127-129.

5 Results

Each monthly dataset accounts for 400-450 thousand bids, the 20-25% of the total amount of offers. Bids are ranked according to the merit order (the price descending order); rejected bids are included as well in order to identify the lower portion of demand curve, where prices are lower than the clearing price and elasticity represents the response of buyers with the lowest willingness to pay. The aggregation of all inelastic bids (bids with no price specification) represents instead the demand intercept. The remaining downward sloping demand curve is derived by horizontally summing up bids characterized by the same price. At the end of the procedure the dataset sample size ranges from 15558 observations in February to 23148 observations in November. For each hour, the posterior distribution of parameters is derived using the full conditional distributions and algorithm described in the previous section.[15] The point estimates of β parameters are given by the posterior means. In order to have some statistical summaries, for each month estimates are aggregated by hour.
Elasticity values are expressed in absolute terms, given that demand elasticity is usually negative. Therefore, "higher elasticity" states for higher absolute value even if the algebraic number is more negative and therefore "lower". Firstly, in 2011 estimates ranged from a minimum value of -0.1434, recorded in September, to -0.0359 recorded in November. Secondly, elasticity varied within the day (Table 2). Peak hour elasticities were, on average, higher than off-peak ones and showed higher variability, going from -0.1434 to -0.0484, while off-peak elasticities varied between -0.070 and -0.0359. This pattern is even clearer considering the average elasticity values aggregated by peak and off-peak hours: the average peak elasticity was -0.0732, while the average off-peak one was -0.052, meaning that electricity demand was more responsive to changes in price during the hours. During peak hours, when business activities were prevalent, the quantities traded larger and congestion more frequent, industrial buyers were able to postpone their consumption, reschedule their demand profiles, and reduce their load levels.
Results suggest another important implication. During peak hours, the range of energy sources deployed is wider, due to the presence of photovoltaic power generation systems injecting electricity in the grid supply. It is plausible to think that a wider set of energy sources involves the availability of more competing substitutes, that increased the demand response to changing price. That means that in 2011 the penetration of RES has been meaningful to increase the competition and market efficiency. Alternatively, during off-peak hours when electricity domestic uses are higher, the lower levels of elasticity denoted the difficulty of postponing consumption and

[15] The whole computation has been performed using MatLab.

Table 2. Hourly Elasticity Estimates. Means of Posterior Distributions.

hh	January	February	March	April	May	June	July	August	September	October	November	December	Mean
							Peak						
9	-0.0756	-0.0699	-0.0869	-0.0676	-0.0853	-0.0495	-0.0506	-0.073	-0.1388	-0.0579	-0.0603	-0.0599	-0.0729
10	-0.0764	-0.0696	-0.0864	-0.0679	-0.0855	-0.0492	-0.0509	-0.0731	-0.1321	-0.0582	-0.0598	-0.06	-0.0724
11	-0.0771	-0.0695	-0.0867	-0.0682	-0.0859	-0.0484	-0.051	-0.0769	-0.1387	-0.0582	-0.0599	-0.0579	-0.0732
12	-0.0771	-0.0705	-0.0872	-0.0682	-0.0863	-0.0485	-0.0504	-0.0728	-0.1342	-0.0578	-0.06	-0.0556	-0.0724
13	-0.0769	-0.0707	-0.0877	-0.0677	-0.0868	-0.049	-0.0505	-0.0754	-0.1355	-0.0568	-0.0599	-0.0571	-0.0728
14	-0.0761	-0.0701	-0.0876	-0.0686	-0.0863	-0.0493	-0.0503	-0.0777	-0.1416	-0.0569	-0.0603	-0.0579	-0.0736
15	-0.0759	-0.0701	-0.0878	-0.0677	-0.0859	-0.0498	-0.0505	-0.0758	-0.1374	-0.0571	-0.0607	-0.0615	-0.0734
16	-0.0758	-0.0706	-0.0875	-0.0682	-0.0852	-0.0496	-0.0508	-0.0756	-0.1398	-0.0578	-0.0602	-0.0588	-0.0733
17	-0.0751	-0.0707	-0.0872	-0.0678	-0.0853	-0.0488	-0.0512	-0.0757	-0.1371	-0.0579	-0.0605	-0.061	-0.0732
18	-0.0751	-0.0706	-0.0868	-0.0676	-0.0859	-0.0486	-0.0514	-0.0767	-0.1369	-0.0578	-0.0611	-0.0612	-0.0733
19	-0.0759	-0.0703	-0.0869	-0.0679	-0.0856	-0.0493	-0.052	-0.0795	-0.1387	-0.0586	-0.061	-0.0613	-0.0739
20	-0.0753	-0.071	-0.0869	-0.0688	-0.0856	-0.0499	-0.0516	-0.0782	-0.1434	-0.0582	-0.0611	-0.0581	-0.0740
Mean	-0.0760	-0.0703	-0.0871	-0.0680	-0.0858	-0.0492	-0.0509	-0.0759	-0.1379	-0.0578	-0.0604	-0.0592	-0.0732
							Off-Peak						
21	-0.0571	-0.0528	-0.0452	-0.058	-0.063	-0.0438	-0.0594	-0.0577	-0.0498	-0.0707	-0.0395	-0.0607	-0.0548
22	-0.0572	-0.0541	-0.0456	-0.0592	-0.0657	-0.0434	-0.0588	-0.0576	-0.0487	-0.0706	-0.0419	-0.0612	-0.0553
23	-0.0573	-0.0556	-0.0489	-0.058	-0.0629	-0.0432	-0.0592	-0.0576	-0.0495	-0.0676	-0.0447	-0.0596	-0.0553
24	-0.0576	-0.0522	-0.0459	-0.0613	-0.0662	-0.0427	-0.0588	-0.0577	-0.0498	-0.0664	-0.0418	-0.0595	-0.0550
1	-0.0576	-0.0553	-0.0455	-0.0595	-0.065	-0.0432	-0.0587	-0.0578	-0.0496	-0.0701	-0.0422	-0.0601	-0.0554
2	-0.0581	-0.0557	-0.047	-0.0603	-0.0679	-0.0439	-0.0588	-0.0578	-0.0503	-0.0675	-0.046	-0.0599	-0.0561
3	-0.058	-0.0559	-0.0504	-0.0619	-0.0632	-0.0438	-0.0588	-0.0577	-0.0514	-0.0645	-0.0372	-0.06	-0.0552
4	-0.0579	-0.0556	-0.0521	-0.0641	-0.0642	-0.0439	-0.059	-0.0577	-0.0506	-0.0668	-0.0365	-0.0604	-0.0557
5	-0.0573	-0.0532	-0.0526	-0.0594	-0.0665	-0.0429	-0.0593	-0.0569	-0.0504	-0.0622	-0.0359	-0.0607	-0.0548
6	-0.0567	-0.0519	-0.0517	-0.0584	-0.0635	-0.0429	-0.0591	-0.0569	-0.051	-0.0675	-0.0362	-0.0603	-0.0547
7	-0.057	-0.0531	-0.05	-0.0582	-0.0666	-0.0435	-0.0593	-0.0569	-0.0509	-0.0689	-0.036	-0.0609	-0.0551
8	-0.0572	-0.0526	-0.0475	-0.0624	-0.0676	-0.0441	-0.0591	-0.0565	-0.0513	-0.0675	-0.0366	-0.0611	-0.0553
Mean	-0.0574	-0.0540	-0.0485	-0.0601	-0.0652	-0.0434	-0.0590	-0.0574	-0.0503	-0.0675	-0.0395	-0.0604	-0.0552
Mountly Mean	-0.0667	-0.0622	-0.0678	-0.0640	-0.0755	-0.0463	-0.0550	-0.0666	-0.0941	-0.0626	-0.0500	-0.0598	-0.0642

changing end-users' habits that made demand stiffer.
Table 2 also aggregates posterior means according to the month (the last row). Elasticity estimates appear to be relatively higher in the winter and lower in the summer months.[16] In the winter months (such as January, February and December) values were around -0.062 and -0.066 while in July values fell to -0.054. This can be explained by the fact that in Summer industry is generally on holiday and consumers have less elastic habits when the season requires air conditioning.
Elasticity values were also differentiated according to the presence of congestion (Table 3). The first two columns show elasticities and frequencies referring to markets characterized by the absence of congestion (one zone), while the third and the fourth columns show elasticities and frequencies recorded when the market was split in two zones, and so on, up to four zones. Aggregation by zone segmentation shows that when single market occurred, elasticity was on average lower than that recorded when market was split in more zones. The average value under single market was -0.059, and the value increased as the number of transmission congestion increased. This suggests that consumers were more responsive when market participation and volume trades were larger. However, if we look at market segmentation differentiating between peak and off-peak hours, a composite behaviour emerges. During peak hours, elasticity kept to be lower when single market occurred, while the off-peak elasticity was lower under congestion. Peak elasticity went to -0.057 under single market to -0.069 under the maximum segmentation in four zones. Off-peak hours recorded instead the highest elasticity of -0.061 under single market, reaching the lowest elasticity under four market segmentation. During off-peak hours, domestic consumption was more prevalent and captive consumers were less sensitive to modify their consuming profiles according to larger forecasted market participation.

[16] This pattern has also been reported, among many others, by Parti and Parti (1980), Mountain (1993),Dubin (2014).

Table 3. Hourly Elasticity Estimates by Zone Congestions.

	Zone							
	1		2		3		4	
	Elasticity	Freq.	Elasticity	Freq.	Elasticity	Freq.	Elasticity	Freq.
	Peak							
January	-0.077	5	-0.077	91	-0.075	269	-0.077	7
February	-0.061	11	-0.061	146	-0.06	178	-0.059	1
March	-0.091	4	-0.085	125	-0.088	226	-0.09	17
April	-0.065	1	-0.067	110	-0.068	215	-0.071	34
May	-0.079	16	-0.084	107	-0.088	225	-0.081	24
June	-0.035	17	-0.029	133	-0.034	205	-0.043	5
July	-0.065	22	-0.06	152	-0.062	196	-0.076	2
August	0	0	-0.067	52	-0.06	311	-0.088	9
September	-0.047	10	-0.147	221	-0.132	124	-0.074	5
October	-0.056	7	-0.06	146	-0.056	202	-0.064	17
November	-0.061	1	-0.061	163	-0.06	156	-0.059	40
December	-0.054	2	-0.06	137	-0.059	202	-0.057	31
Mean	-0.058	8	-0.072	132	-0.070	209	-0.070	16
	OffPeak							
January	-0.055	7	-0.057	117	-0.057	121	-0.058	127
February	-0.048	10	-0.052	73	-0.057	135	-0.053	118
March	-0.07	3	-0.053	105	-0.042	122	-0.05	141
April	-0.058	8	-0.06	152	-0.061	120	-0.06	80
May	-0.067	15	-0.061	162	-0.07	165	-0.061	30
June	-0.04	18	-0.035	123	-0.051	157	-0.043	62
July	-0.057	25	-0.058	193	-0.06	109	-0.062	45
August	-0.079	5	-0.057	162	-0.06	128	-0.051	77
September	-0.059	7	-0.054	172	-0.047	99	-0.046	82
October	-0.067	7	-0.066	93	-0.069	155	-0.067	117
November	-0.079	1	-0.057	94	-0.06	127	-0.051	138
December	-0.048	4	-0.052	67	-0.057	153	-0.053	148
Mean	-0.061	9	-0.055	126	-0.058	133	-0.055	97
Overal Mean	-0.059	9	-0.063	129	-0.064	171	-0.062	57

Lastly, elasticity was different according to the different clearing prices. Table 4 shows summary statistics of PUN's distribution for the whole 2011. The mean value of first decile was 44.40 euro/MWh and that of the last decile was 97.67 euro/MWh.

Table 4: System Marginal Prices by Deciles: euro/Mwh.

Decile	Min	Average	Max
1	10	44.4	51.21
2	51.25	55.31	58.97
3	58.98	61.59	63.79
4	63.79	65.66	67.19
5	67.19	68.44	69.7
6	69.7	71.06	72.61
7	72.61	74.2	75.92
8	75.93	78.03	80.63
9	80.64	83.75	87.57
10	87.6	97.67	142.96
Mean	63.769	70.011	77.055

Looking at Table 5, where elasticity is aggregated by PUN deciles, on average, peak values were higher than off-peak values. Peak elasticity ranged between -0.063 in the first decile to -0.075 in the last decile, while off-peak one showed less variability, ranging from -0.054 in the first decile to -0.058 in the 10th decile. This should suggest a regular downward trend from the 10th to the first decile, nevertheless, the highest value of -0.076 (in the 5th decile of peak rows) and -0.058 (in the 7th decile of off-peak rows) were recorded in the middle of the PUN's distribution. Overall, the empirical analysis shows that elasticity of demand varied according to several features of market structure. Firstly, it is plausible to conclude that price elasticities were higher when the availability of competing substitutes to electricity was larger. These are situations in which, for instance, Italian consumers use electricity for marginal daily operations (peak hours), flexible industrial uses (weekdays), marginal heating (winter). In such situations, consumers show a more flexible, i.e. elastic behaviour.
Secondly, the higher elasticity during peak hours can be correlated even with the dramatic penetration of RES that made wider the range of sources deployed in power generation. Lastly, demand is more elastic when the expenditure is relatively more significant. This explains why elasticity values were relatively higher with higher PUN levels and under transmission congestion, when volume trades are larger.

Table 5. Hourly Elasticity Estimates by PUN's percentiles.

	January	February	March	April	May	Jun	July	August	September	October	November	December	Mean
Decile							Peak						
10	-0.070	-0.071	-0.086	-0.066	-0.083	-0.052	-0.051	-0.084	-0.165	-0.058	-0.061	-0.062	-0.076
9	-0.074	-0.069	-0.086	-0.069	-0.088	-0.052	-0.053	-0.084	-0.121	-0.057	-0.061	-0.060	-0.073
8	-0.074	-0.071	-0.085	-0.071	-0.086	-0.050	-0.049	-0.072	-0.127	-0.059	-0.059	-0.054	-0.071
7	-0.076	-0.069	-0.088	-0.068	-0.088	-0.048	-0.052	-0.075	-0.116	-0.057	-0.061	-0.048	-0.071
6	-0.080	-0.070	-0.086	-0.065	-0.086	-0.049	-0.051	-0.073	-0.081	-0.057	-0.059	-0.063	-0.068
5	-0.076	-0.070	-0.088	-0.069	-0.083	-0.051	-0.051	-0.074	-0.174	-0.060	-0.058	-0.064	-0.077
4	-0.076	-0.070	-0.089	-0.067	-0.084	-0.046	-0.049	-0.071	-0.150	-0.059	-0.062	-0.064	-0.074
3	-0.072	-0.073	-0.090	-0.067	-0.084	-0.050	-0.052	-0.078	-0.161	-0.056	-0.062	-0.063	-0.076
2	-0.072	-0.075	-0.089	-0.072	-0.081	-0.046	-0.047	-0.067	-0.129	-0.061	-0.063	-0.092	-0.074
1	.	.	.	-0.071	-0.080	-0.039	-0.055	-0.069	.	-0.063	-0.064	.	-0.063
							OffPeak						
10	.	.	-0.072	-0.044	-0.074	-0.057	-0.065	-0.048	-0.050	-0.071	-0.038	-0.062	-0.058
9	.	-0.037	-0.050	-0.066	-0.059	-0.044	-0.069	-0.061	-0.050	-0.061	-0.045	-0.062	-0.055
8	-0.069	-0.063	-0.041	-0.061	-0.071	-0.045	-0.060	-0.059	-0.055	-0.077	-0.030	-0.058	-0.058
7	-0.066	-0.051	-0.060	-0.071	-0.073	-0.045	-0.058	-0.056	-0.048	-0.065	-0.047	-0.061	-0.059
6	-0.056	-0.051	-0.059	-0.059	-0.068	-0.047	-0.061	-0.053	-0.044	-0.070	-0.041	-0.057	-0.056
5	-0.054	-0.056	-0.030	-0.060	-0.073	-0.047	-0.065	-0.056	-0.048	-0.067	-0.043	-0.064	-0.055
4	-0.055	-0.052	-0.043	-0.057	-0.058	-0.040	-0.060	-0.062	-0.051	-0.064	-0.038	-0.070	-0.054
3	-0.058	-0.056	-0.050	-0.061	-0.063	-0.045	-0.058	-0.056	-0.051	-0.056	-0.052	-0.066	-0.056
2	-0.061	-0.050	-0.054	-0.062	-0.066	-0.040	-0.061	-0.064	-0.056	-0.067	-0.043	-0.053	-0.056
1	-0.055	-0.057	-0.048	-0.059	-0.057	-0.043	-0.053	-0.060	-0.054	-0.076	-0.030	-0.061	-0.054

6 Concluding Remarks

This paper performed a Bayesian estimation of price elasticity of electricity demand for the Italian DAM. Unlike previous studies, particularly focused on aggregated market data, this study adopted a microeconomic perspective using the ex-ante demand bids that represent the real willingness to pay of buyers. The duality approach was then applied to shift from the microeconomic optimization problem to the Marshallian demand. The empirical demand for electricity was computed by horizontally summing up the bids with the same price. The novel feature of this study is the use of Bayesian method that allowed model to be more flexible and compute more robust estimates. The Bayesian framework treated parameters indexing the data generating process as random variables. This randomness expresses the uncertainty about the true values of parameters and was modelled through the prior distribution that exploited the pre-experimental information. Within an elegant analytic framework, prior distributions were shaped according to past inference results and the institutional structure governing the agent behaviours (i.e. he presence of dual pricing scheme). The preliminary hypotheses on parameters' values were then updated according to the observed data. Moreover, homoskedasticity assumption was relaxed in favour of two more realistic assumptions. First, agents' price response can change under time-varying pricing scheme and can be correlated within the two periods of the day (9a.m.-8p.m and 9p.m.-8a.m). Second, the price responses are heteroskedastic, that is, they can be different among hours. Dimensionality problems were overcome through the exchangebility property that made the framework flexible but enough structured for statistical inference. Exchangebility has allowed in fact for individuals' error terms being different but drawn from the same distribution, making the model hierarchical.
Inference showed essentially three main results; first, there exist a well-defined value for the demand elasticity, which ranges between -0.1434 to -0.0359; second, average elasticity values differ between the peak and off-peak hours and this behaviour is consistent with the dual tariff scheme introduced in 2011. Third, elasticity varies according to some characteristics of market structure such as the presence of congestions and the level of PUN.
Overall, paper provided a statistical framework, as flexible as structured, to reliably estimate the price elasticity of electricity demand. It is expected it helps to understand at the microeconomic level how changes in electricity prices due to new policy and institutional schemes may impact the firm and household demands.

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